

ADAPTIVE SPACETIME MESHING IN $2D \times TIME$ FOR NONLINEAR AND ANISOTROPIC MEDIA

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Scientists and engineers use hyperbolic partial differential equations (PDEs) to model wave propagation phenomena. Anisotropy of the transmitting medium can cause waves to propagate asymmetrically, with different speeds in different directions. Shocks (discontinuities) can form spontaneously when the PDE is nonlinear. In this case, the local wave speeds are nonuniform, solution-dependent functions over the spacetime analysis domain, and the wave speeds at a given spatial location can change discontinuously with time. Spacetime discontinuous Galerkin finite element methods provide efficient and accurate solutions to hyperbolic PDEs when implemented on spacetime meshes that are *causal*, as described below. This paper unifies previous work [1, 2] to define a new adaptive meshing algorithm for nonlinear and anisotropic hyperbolic problems in $2D \times time$. The new algorithm generates causal spacetime meshes comprised of tetrahedral elements with provable worst-case guarantees for the element *temporal aspect ratios*.

Our meshing algorithm constructs an unstructured, nonconforming tetrahedral partition of a given spacetime analysis domain, $\Omega \subset \mathbb{E}^2 \times \mathbb{R}$. The algorithm advances the mesh through a series of triangulated terrains embedded in Ω called *fronts*. A front τ is *causal* if and only if it lies below the *cone of influence* of every point p on τ . The slope in any spatial direction of the cone of influence of p is the reciprocal of the maximum wave speed at p in that direction. The algorithm advances a local neighborhood N of τ to the neighborhood N' of a new causal front τ' ; then it generates new tetrahedra over the spacetime volume between τ and τ' where the solution can be computed immediately.

Our algorithm maximizes the duration of new elements subject to causality of the new front, for which a necessary, *nonlocal* condition is that N' must lie below the lower hull of the domains of influence of all points on $\tau \setminus N$. To compute an estimate of the most constraining domain of influence, we use a discrete *bounding cone hierarchy* induced by a balanced partition of the triangular tessellation of τ to query efficiently the cones of influence on τ . When the underlying medium is *anisotropic*, the cones are asymmetric, e.g., with elliptical cross-sections. When the underlying PDE is nonlinear, a distant but fast wave (estimated by a *nonlocal* cone) can overtake a slower wave and hence limit the duration of new elements.

Our unified algorithm simultaneously adapts the size and—because of the causality constraint—also the duration of future spacetime elements to *a posteriori* estimates of numerical error. If the elements created at any step generate too much error, they are rejected and the front is refined by repeated bisection. The resulting smaller triangles lead to smaller spacetime elements with potentially better temporal aspect ratios. We coarsen the front to generate larger spacetime tetrahedra when the error is sufficiently small.

References

- [1] R. Abedi, S.-H. Chung, J. Erickson, Y. Fan, M. Garland, D. Guoy, R. Haber, J. M. Sullivan, S. Thite, Y. Zhou. Spacetime meshing with adaptive refinement and coarsening. In *Proc. Symp. Comput. Geom.*, pp. 300–309, 2004.
- [2] S. Thite. Efficient spacetime meshing with nonlocal cone constraints. In *Proc. 13th International Meshing Roundtable*, pp. 47–58, 2004.

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