MESHING IN 2D×TIME FOR FRONT-TRACKING DG METHODS

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Spacetime discontinuous Galerkin (SDG) finite element methods are used to solve hyperbolic PDEs describing wavelike phenomena, such as fluid flow. In *front-tracking* SDG methods, the inclined facets of spacetime elements are aligned with the trajectories of moving domain boundaries, phase interfaces, and other singular surfaces. These methods are particularly effective because SDG solution fields naturally accommodate the jumps that occur at singular surfaces. Our goal is to construct front-tracking tetrahedral meshes in 2D×time that partition a spacetime analysis domain $\Omega \subset \mathbb{E}^2 \times \mathbb{R}$ while satisfying a *causality* condition that facilitates the SDG solution procedure. Trajectories of $\partial\Omega$ and of interior singular surfaces are generally solution-dependent and must be computed as the solution evolves.

The algorithm advances the mesh through a series of triangulated terrains embedded in Ω called *fronts*. A front τ is *causal* if and only if it lies below the *cone of influence* of every point P on τ. At each step, our algorithm advances a local neighborhood N of the current front τ to a neighborhood N' of a new causal front τ'. Then it generates new tetrahedra over the spacetime volume between N and N', where the solution is immediately computed. In addition to limits imposed directly by the causality requirement for N', so-called *progress constraints* limit the temporal progress at each step to ensure that the spacetime tetrahedra created in the next step are nondegenerate [1]. The progress constraints are complicated because they involve the spatial aspect ratios of triangles in the new front τ'' of the *next* step. Thus, the challenge we confront is to anticipate and process future changes in the front geometry to avoid degenerate or even inverted elements as we track the moving surfaces.

We report recent progress towards a 2D×time front-tracking meshing capability. For some restricted but important cases, we are able to prove bounds on the worst-case *spatial* and *temporal aspect ratios* of spacetime tetrahedra which guarantees that they are nondegenerate. For instance, when the motion of an external boundary or internal singular surface is smooth, or when interfaces move in predictable directions, with discontinuous changes in the motion only at predictable discrete *events*, then the changes in the front geometry are computable in advance. The capability to adapt to these predictable surface motions extends previous work which assumes that the spatial projection of the front τ at any step is a hierarchical refinement of an initial space mesh [2]. We strengthen the progress constraints to anticipate future changes in the front geometry. In addition to adaptive refinement and coarsening, we incorporate other front-modification operations—such as edge flips, edge contractions, and edge dilations—that improve the spatial aspect ratios of front triangles while simultaneously advancing the front in time.

References

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