Efficient Spacetime Meshing with Nonlocal Cone Constraints

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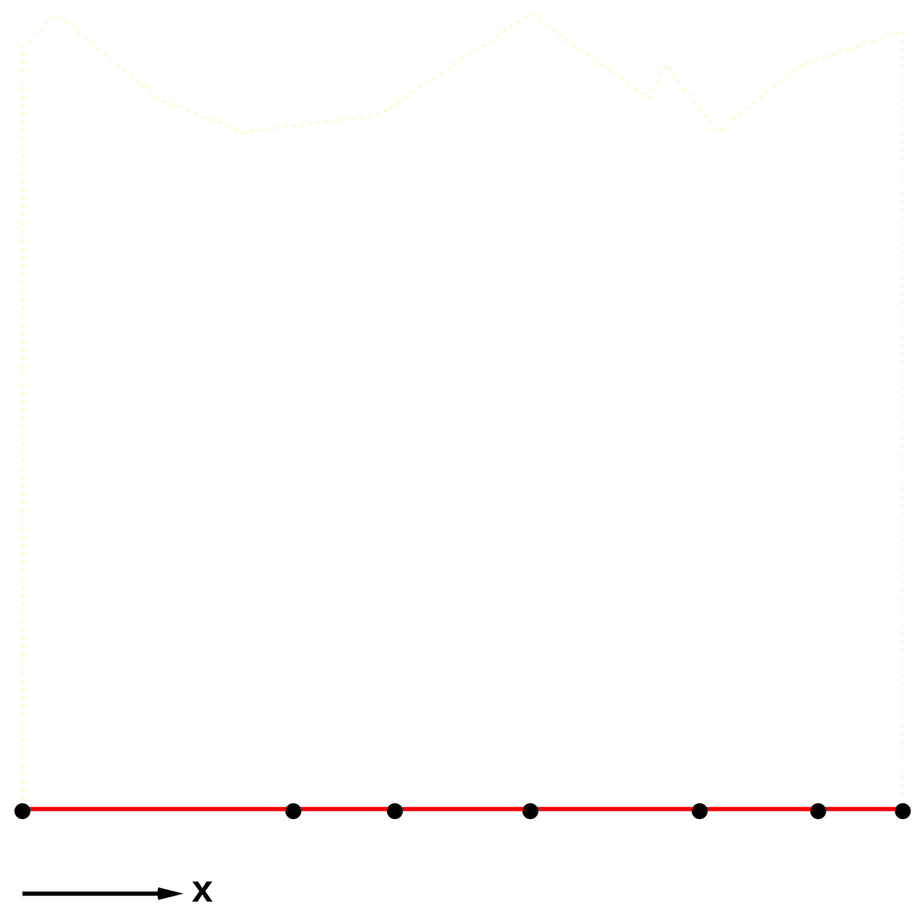
Spacetime Discontinuous Galerkin Methods

- Solve problems involving hyperbolic PDEs
- Solution has wave-like nature
- E.g., ripples on the surface of a pond.
- E.g., gas dynamics.

Spacetime DG Methods

- DG vs. standard FEM: Basis functions defined over elements, discontinuous.
- The solution technique is a time-adaptive method that does not impose a uniform time step.
- Interleave mesh generation and analysis.

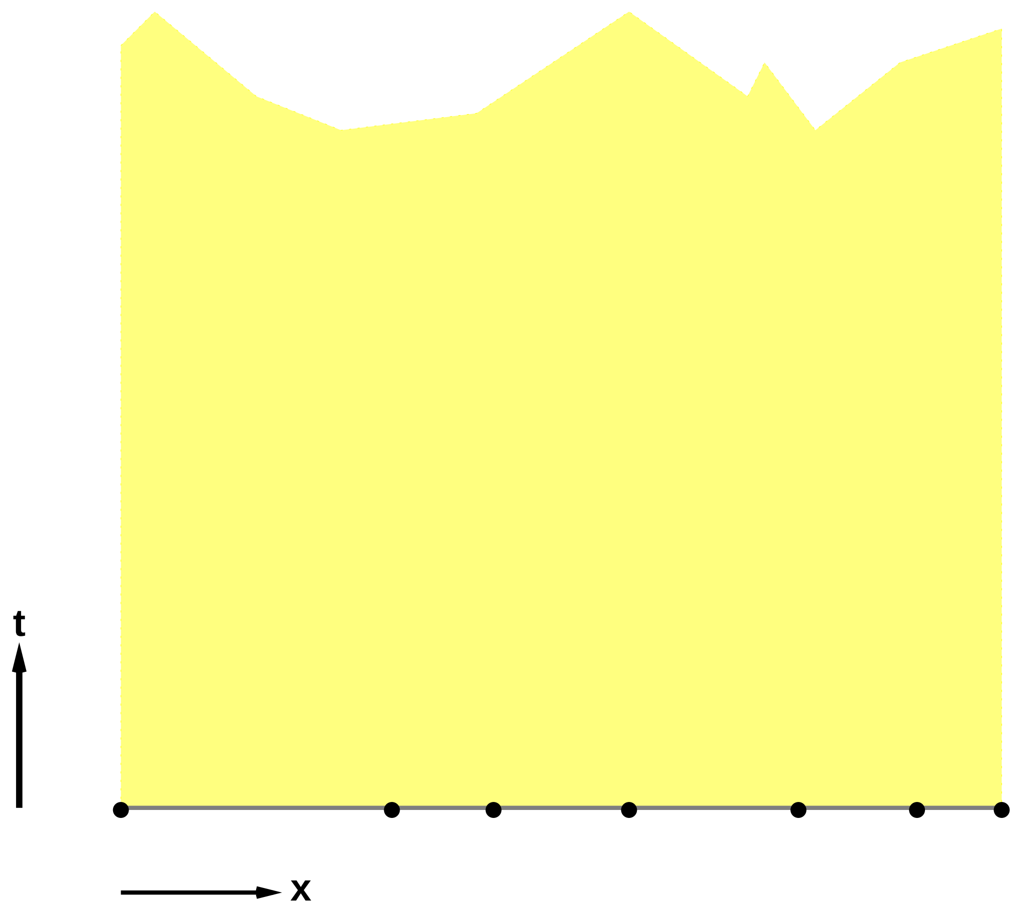
Input



$K \equiv$ a simplicial mesh of the space domain X

$X \subset \mathbb{R}^d$

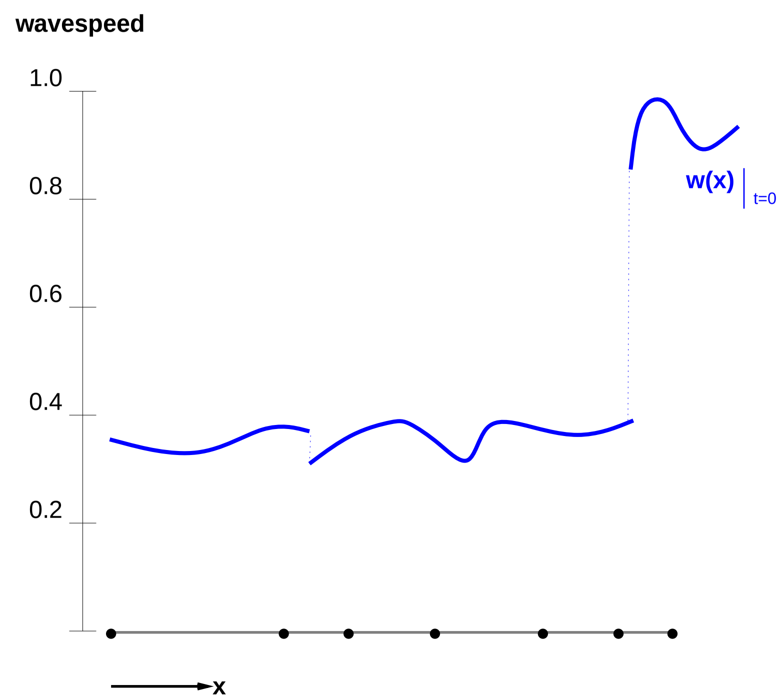
Output



$M \equiv$ a simplicial mesh of $\Gamma = X \times [0, +\infty)$

$\Gamma \subset \mathbb{R}^{d+1}$

Wavespeed

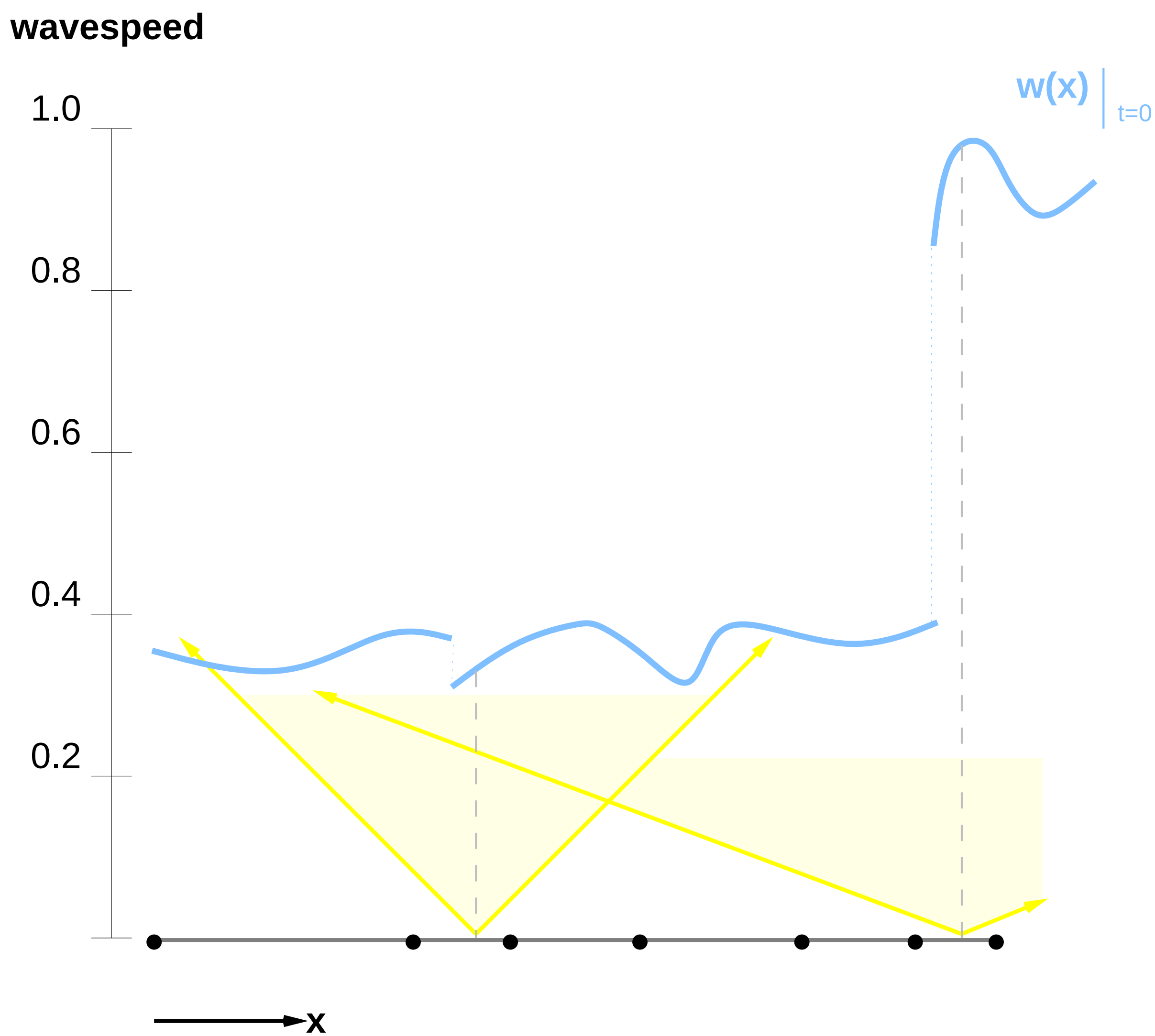


$$w : \Gamma \to \mathbb{R} \text{ such that}$$
$$\forall p \in \Gamma, 0 < w(p) \leq c_{\text{max}}$$

Wavespeed is

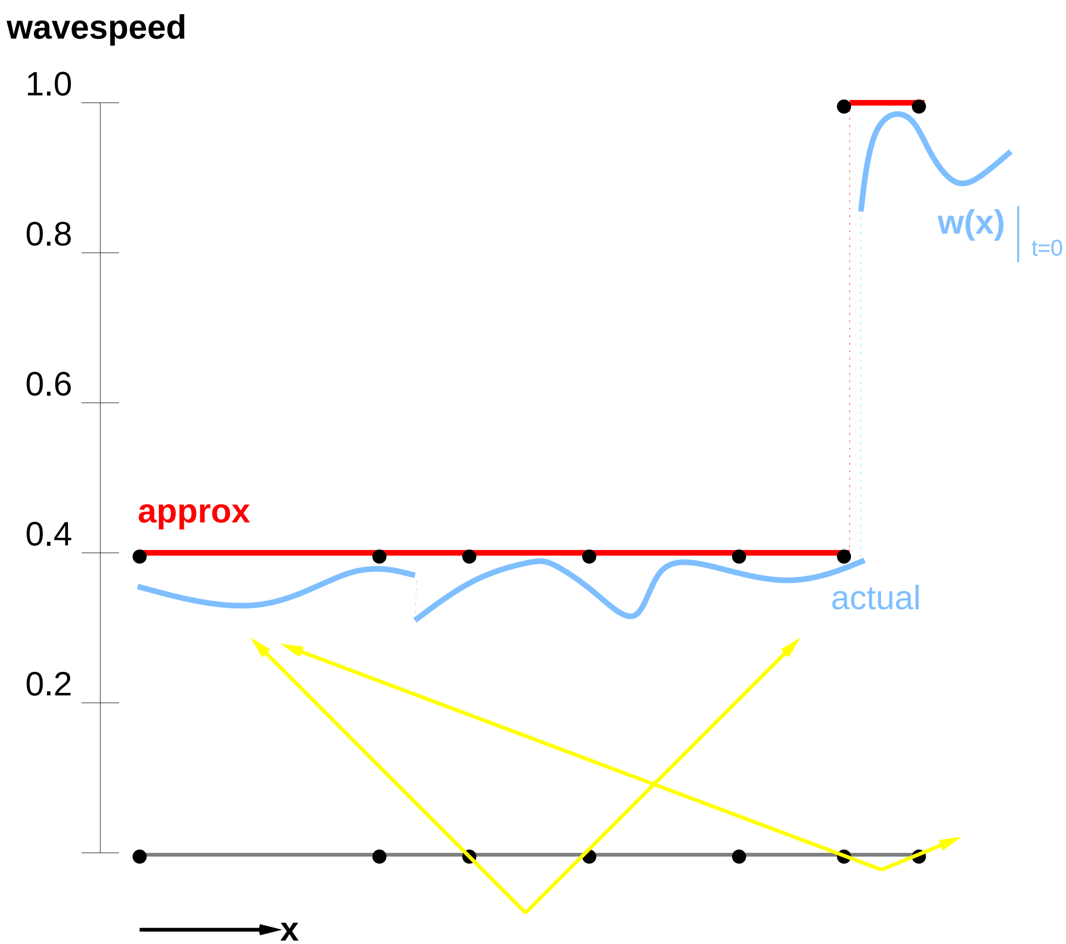
- not constant (depends on the solution)
- computed as the mesh is generated

Cones of influence



A point influences only other points in its cone of influence.

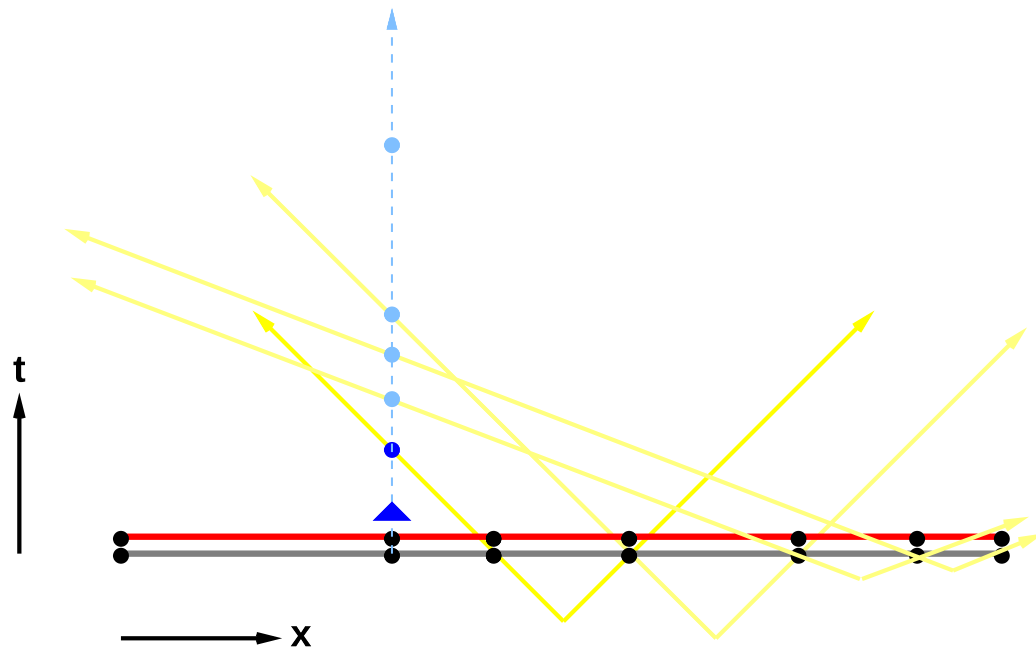
Conservative approximation of wavespeed



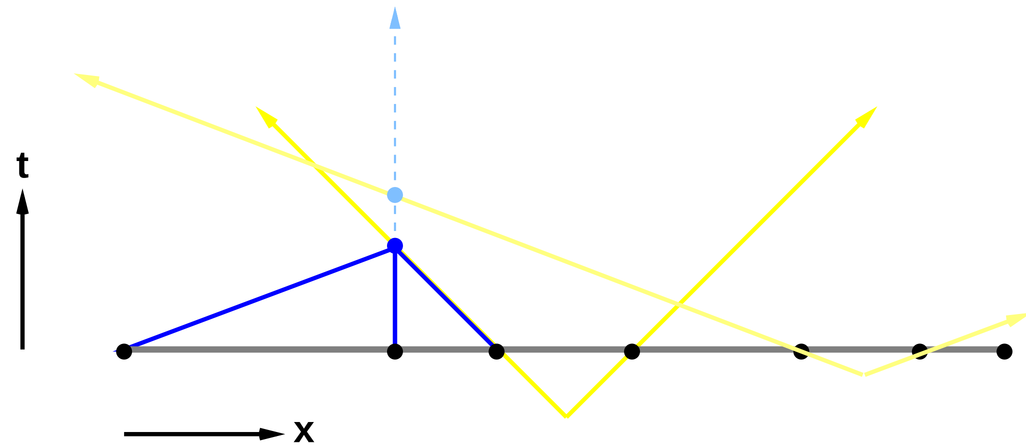
Piecewise constant

Bounding cone for each element contains cone of influence for every point of the element.

Lifting a local minimum



Tent pitching

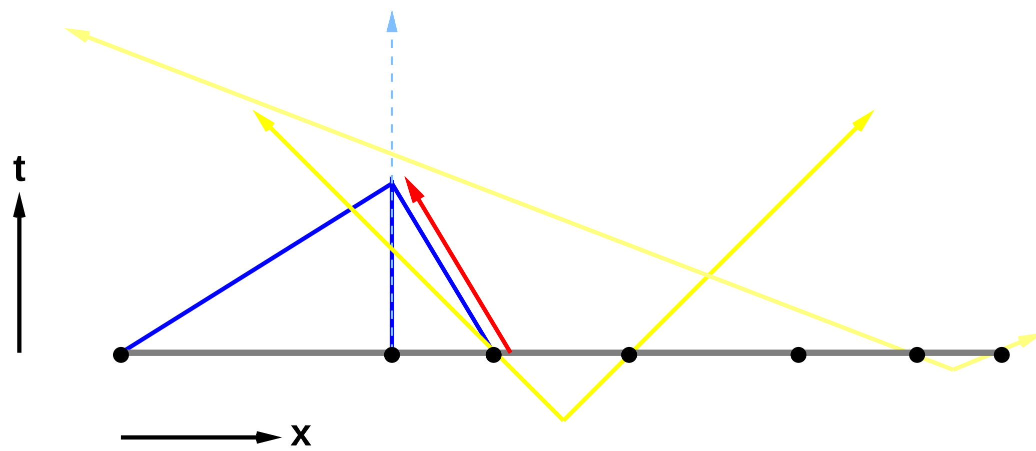


Patch $\equiv$ set of elements in the tent

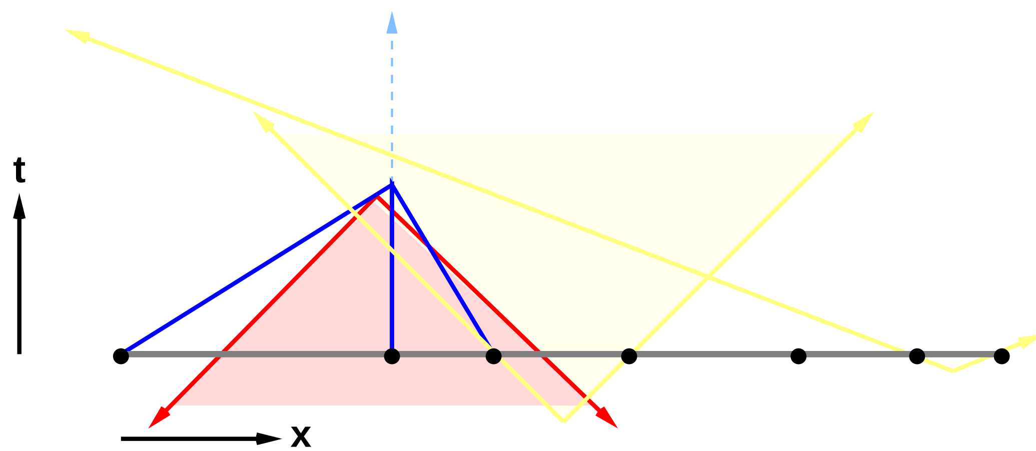
Mesh constructed by incrementally adding patches and solving each patch.

Interdependence between patches is a partial order.

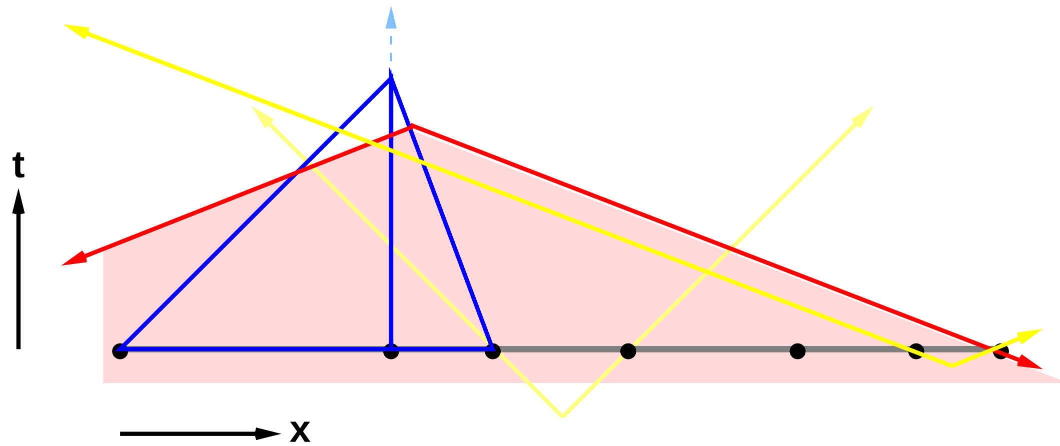
Violated cone constraint



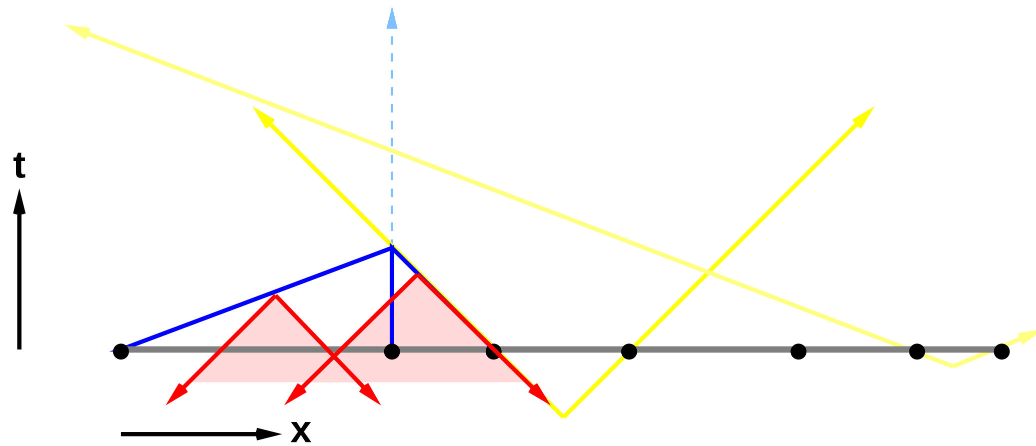
Violated cone constraint (2)



Violated cone constraint (3)



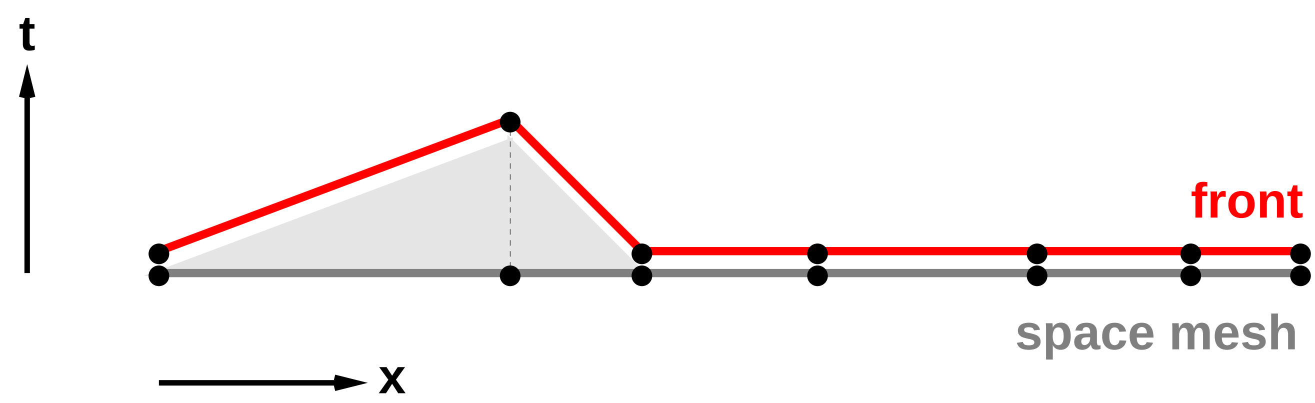
Cones of dependence



A point depends only on other points in its cone of dependence

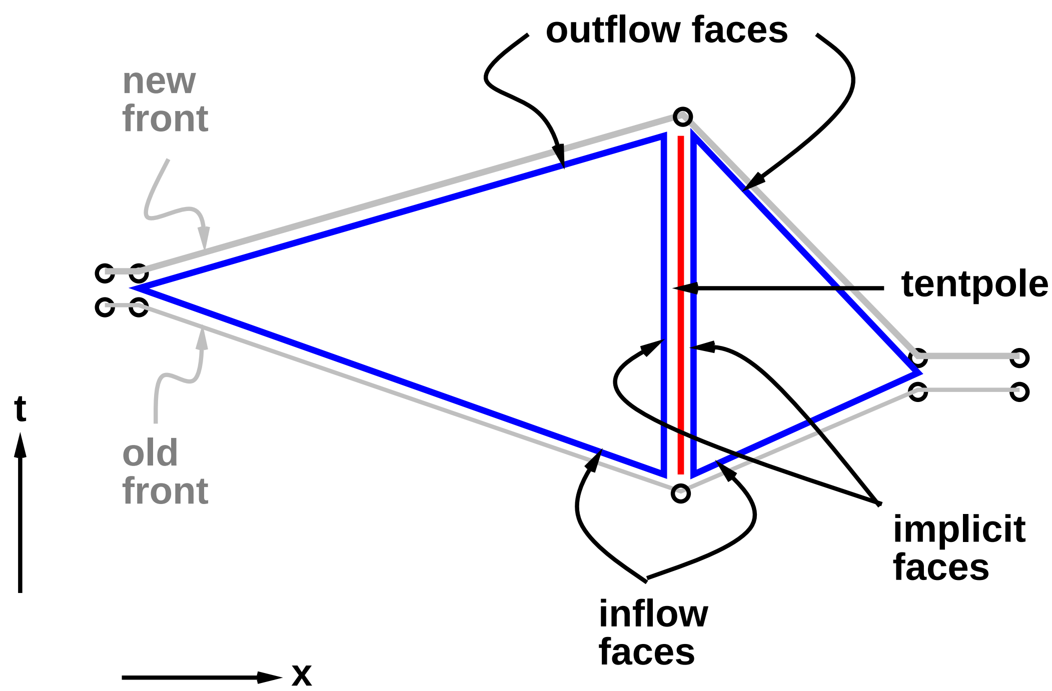
Elements in patch are coupled and must be solved simultaneously

Advancing front



Front $\equiv$ a piecewise linear terrain
whose spatial projection is identical to the space mesh.

Anatomy of a patch

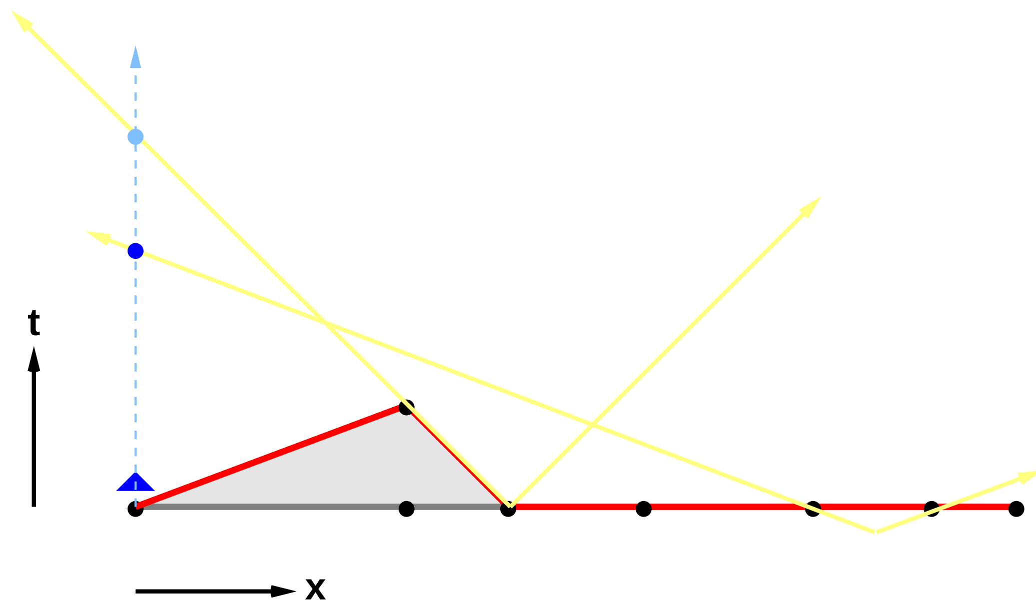


Inflow and outflow faces satisfy cone constraints.

A patch can be solved as soon as it is created.

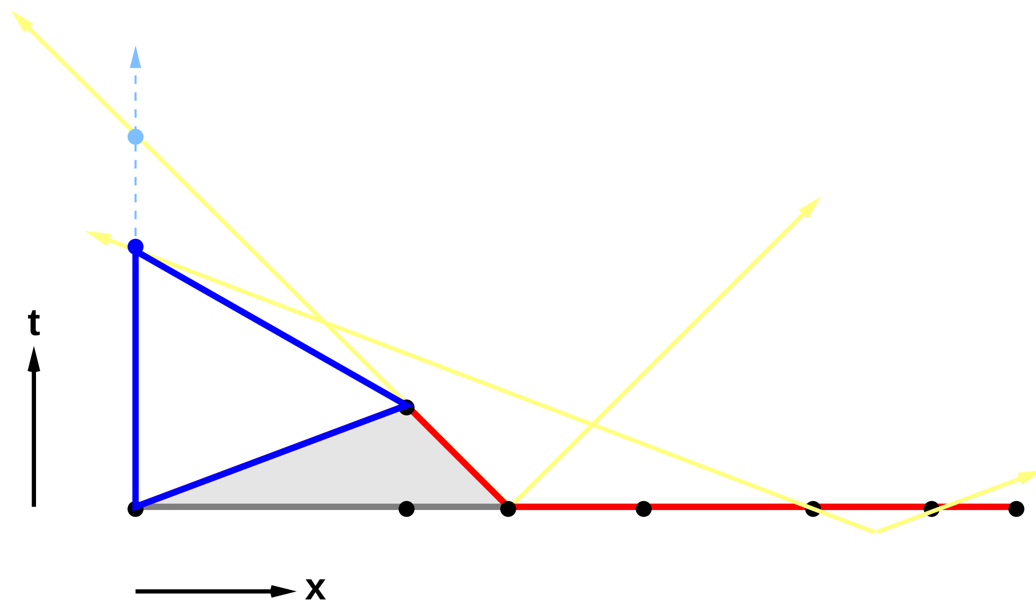
Derive cone constraints for the new outflow faces.

Lifting a local minimum (2)

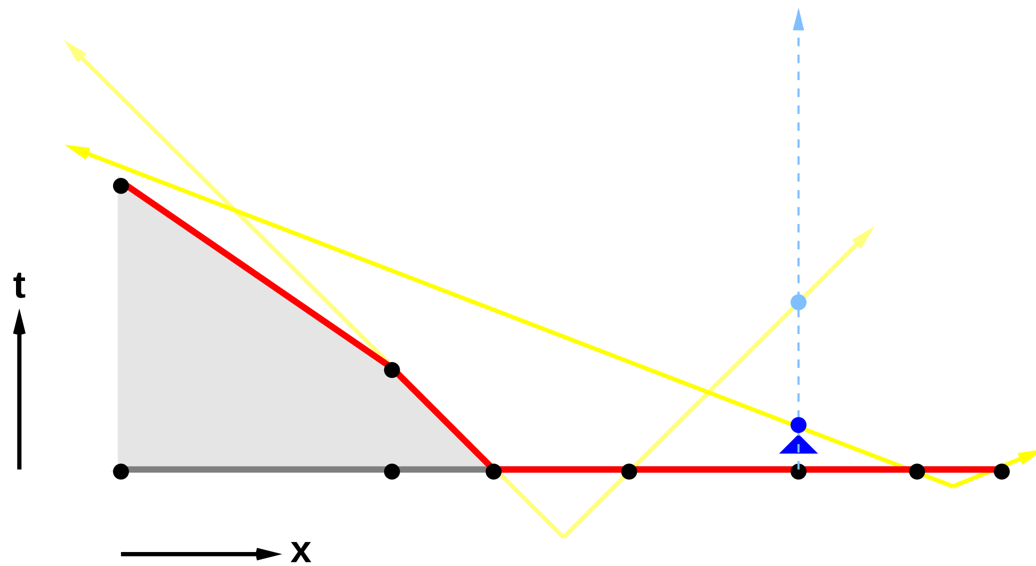


Cone constraints come from outside the patch.

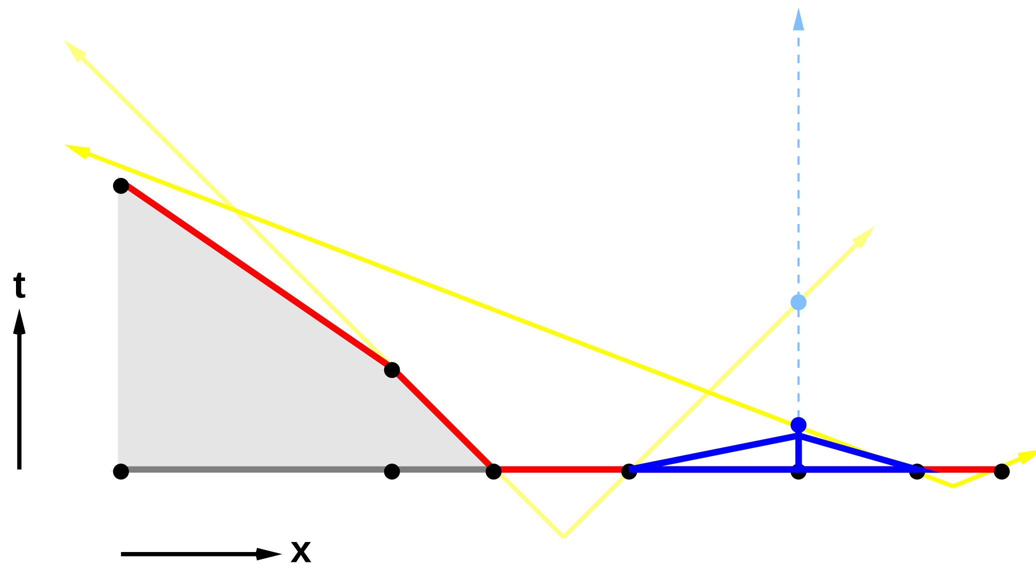
Lifting a local minimum (2)



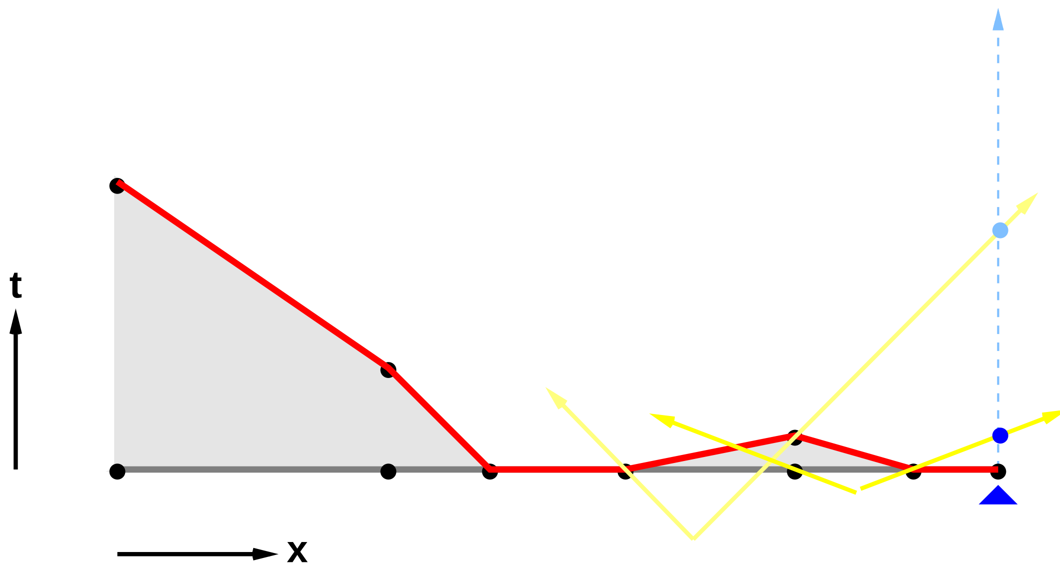
Lifting a local minimum (3)



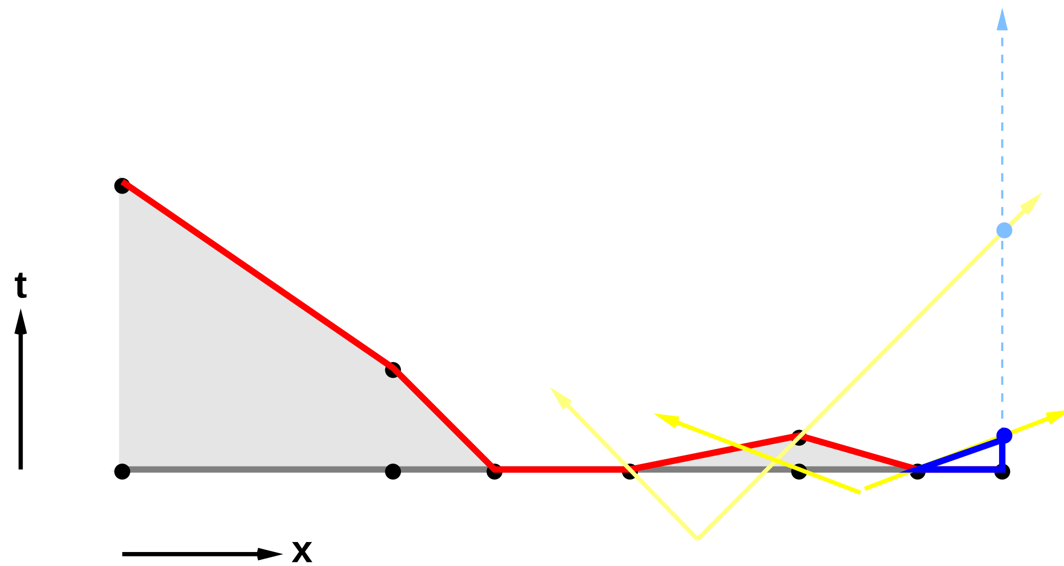
Lifting a local minimum (3)



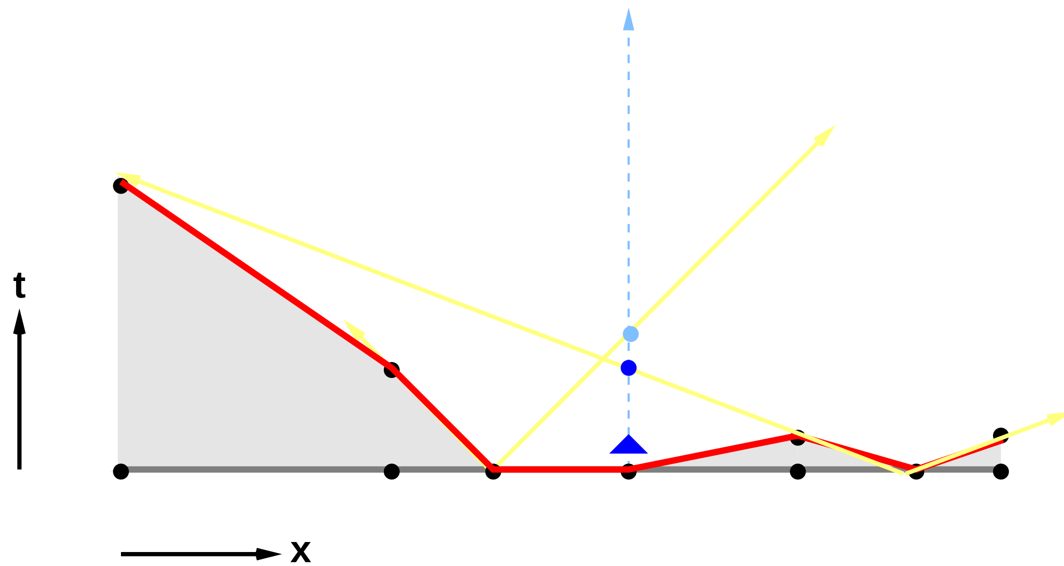
Lifting a local minimum (4)



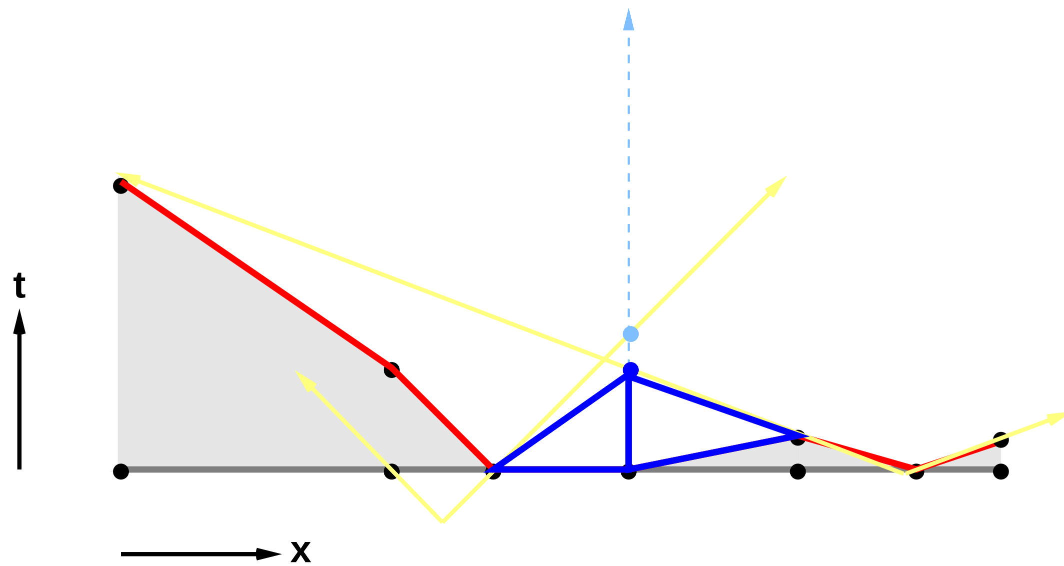
Lifting a local minimum (4)



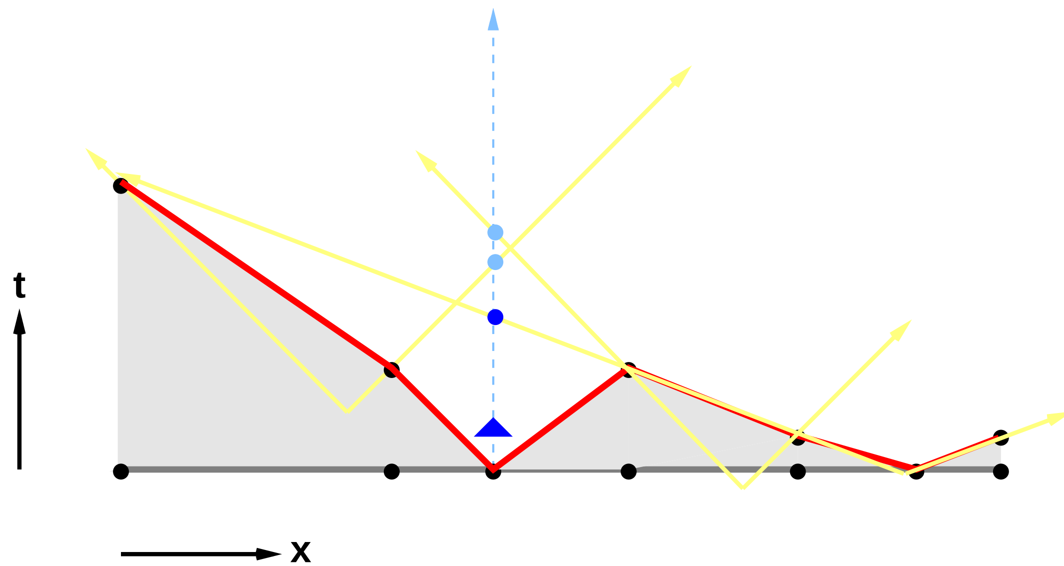
Lifting a local minimum (5)



Lifting a local minimum (5)

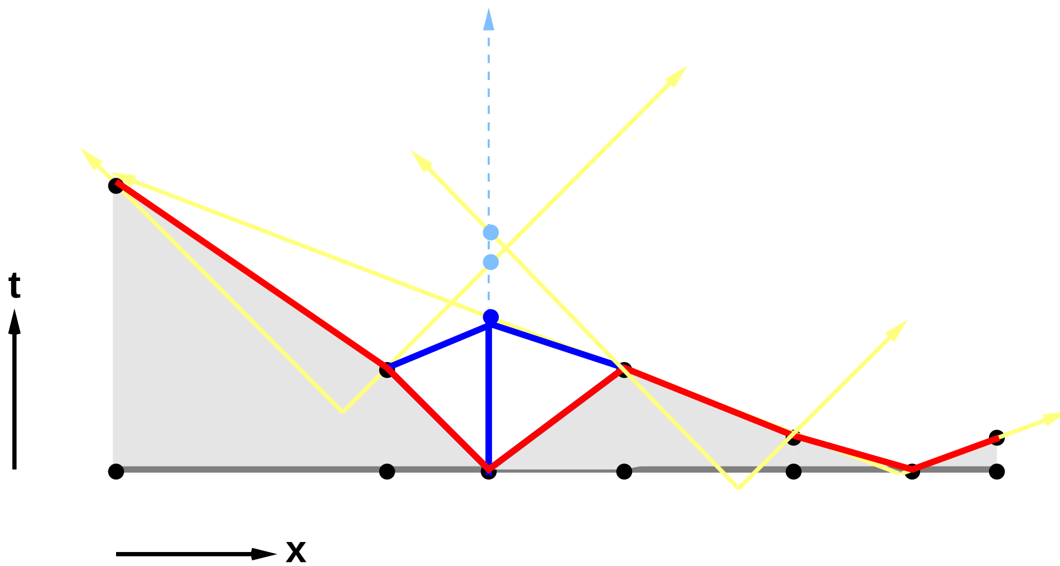


Lifting a local minimum (6)

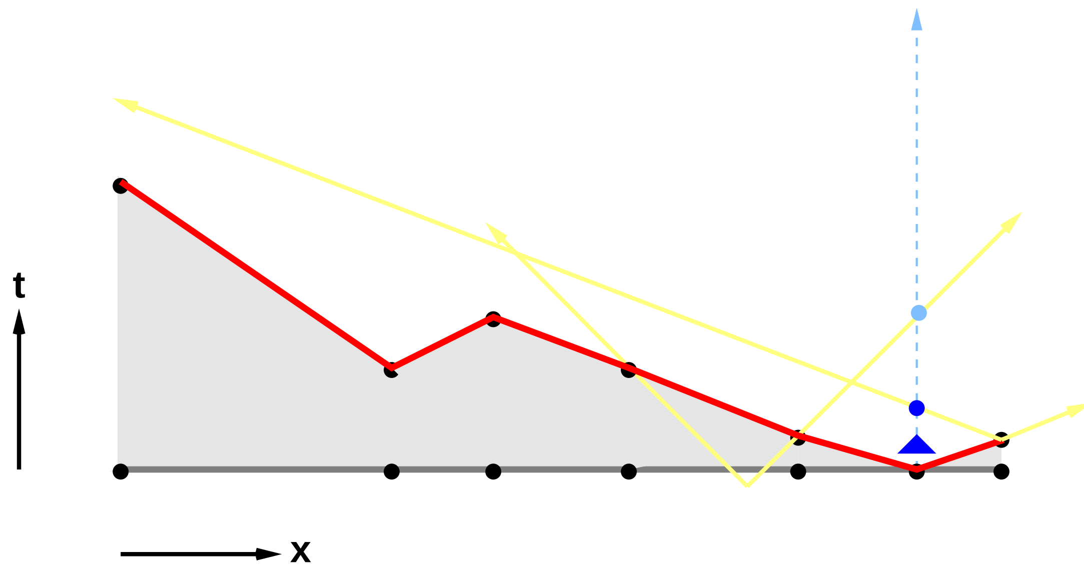


Strictest cone constraint is nonlocal.

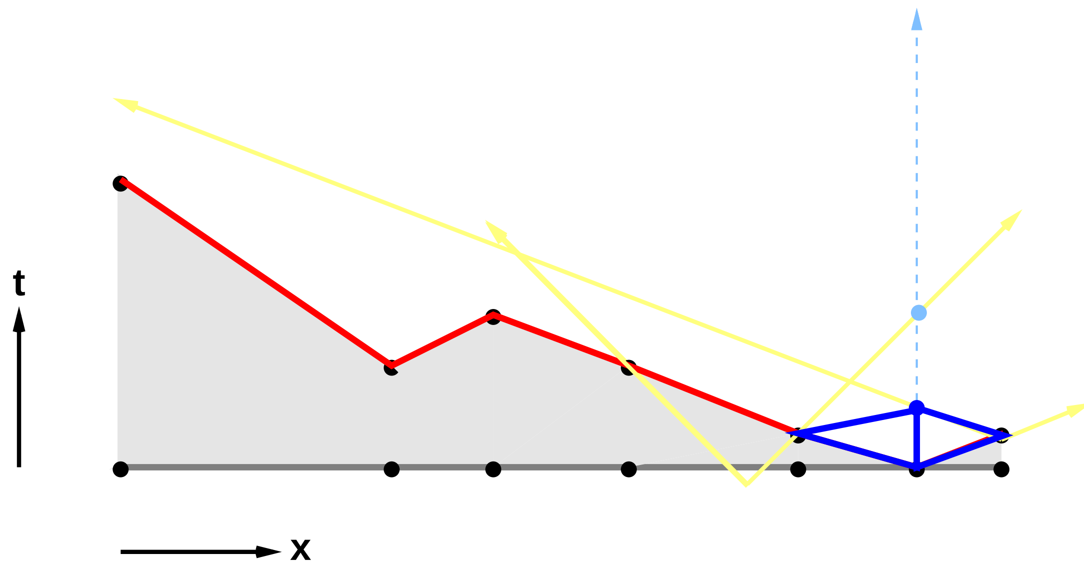
Lifting a local minimum (6)



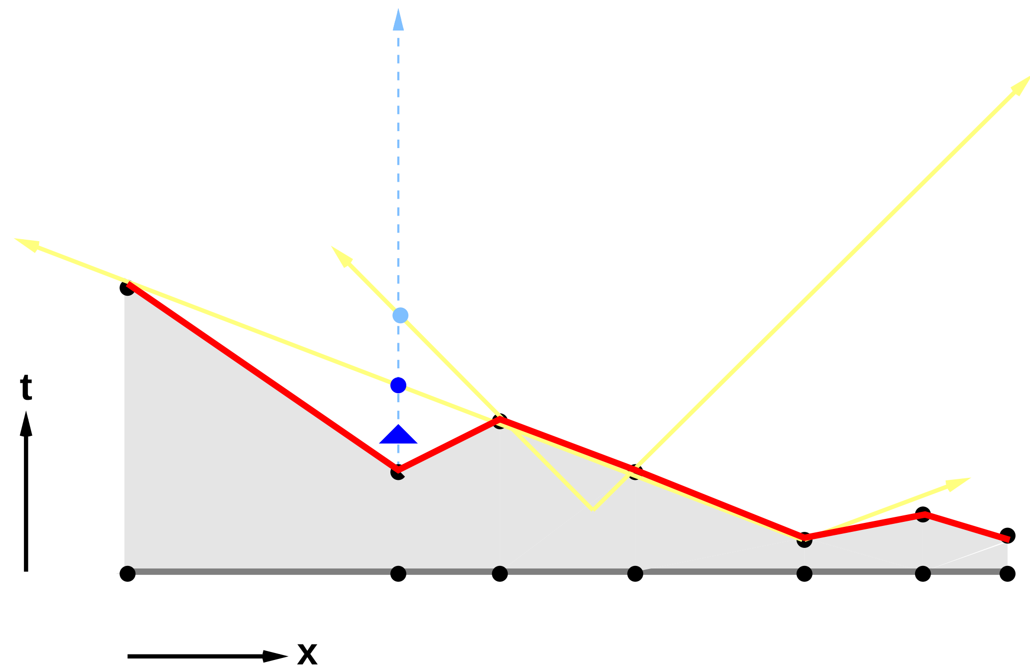
Lifting a local minimum (7)



Lifting a local minimum (7)

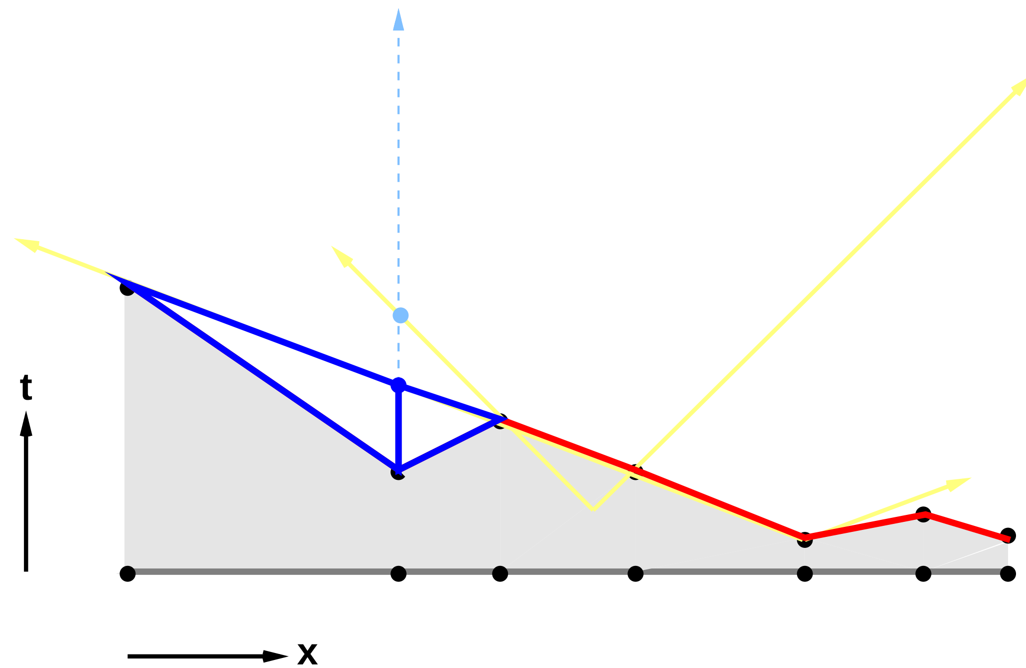


Lifting a local minimum (8)

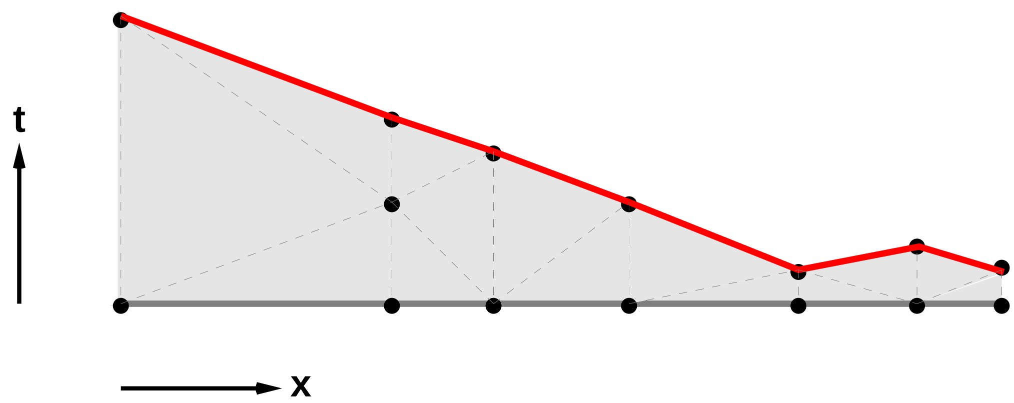


Global minimum has advanced.

Lifting a local minimum (8)



Lifting a local minimum (8)



- Spatial and temporal size of an element adapts to
  - local size of space mesh
  - changing wavespeed

Previous work

Building Spacetime Meshes over Arbitrary Spatial Domains
Jeff Erickson, Damrong Guoy, John M. Sullivan, and Alper Üngör
Proc. 11th Int'l. Meshing Roundtable, pp. 391–402, 2002

Tent-Pitcher: A meshing algorithm for space-time discontinuous Galerkin methods
Alper Üngör and Alla Sheffer
Proc. 9th Int'l. Meshing Roundtable, pp. 111–122, 2000

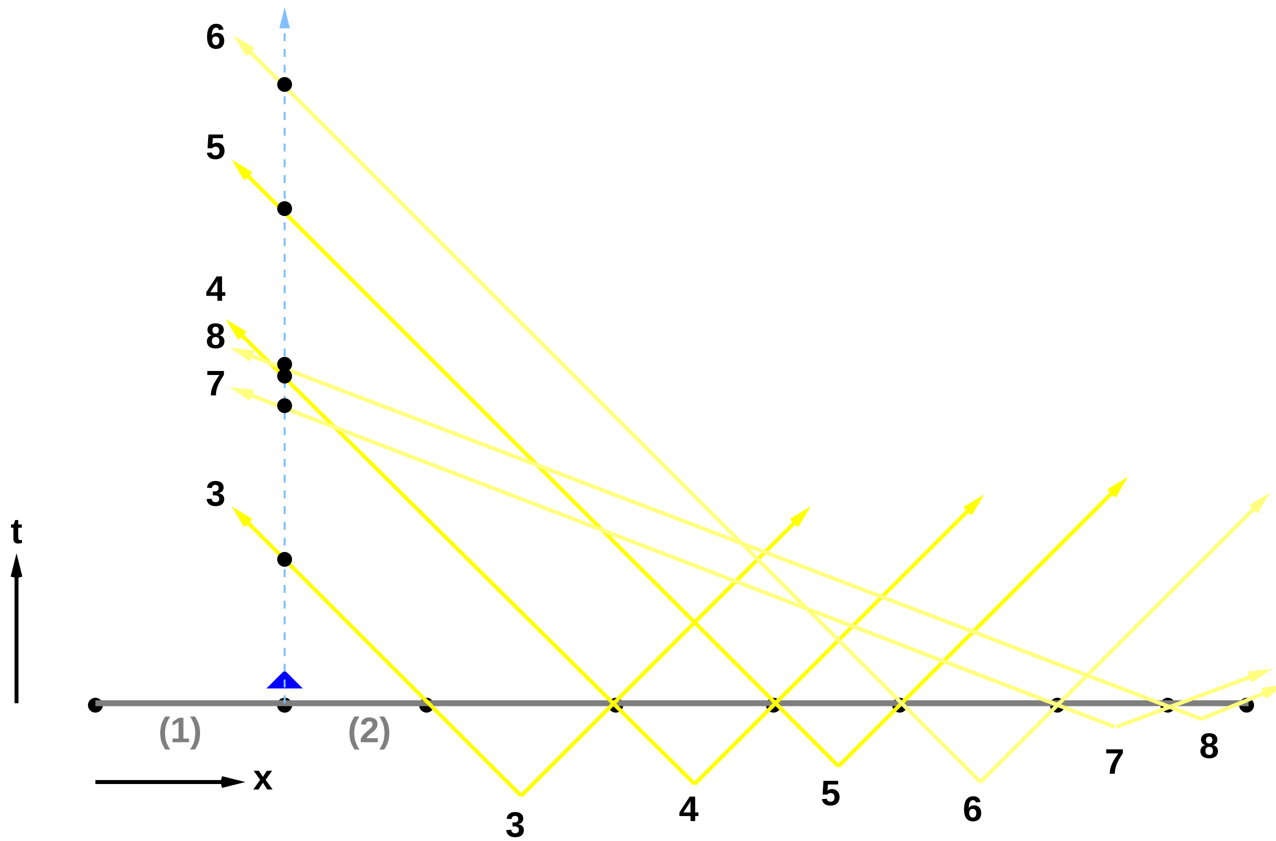
Previous work

- Use the maximum wavespeed at any point in spacetime as a conservative estimate of the wavespeed everywhere.
- Liimited to linear problems with uniform cone constraints.
- It is always possible to advance a local minimum by at least $\varepsilon > 0$, where ε is a constant depending on the space mesh and c_{max}.
- If a simplex in the space mesh contains an obtuse angle, satisfy stronger constraints that guarantee progress.
- For any target time $T > 0$, every vertex on the front has advanced past time T after at most $n\frac{T}{\varepsilon}$ patches.

New results

- We allow cone constraints that vary in spacetime.
- We also handle shocks (discontinuities) in wavespeed.

Bounding cone hierarchy

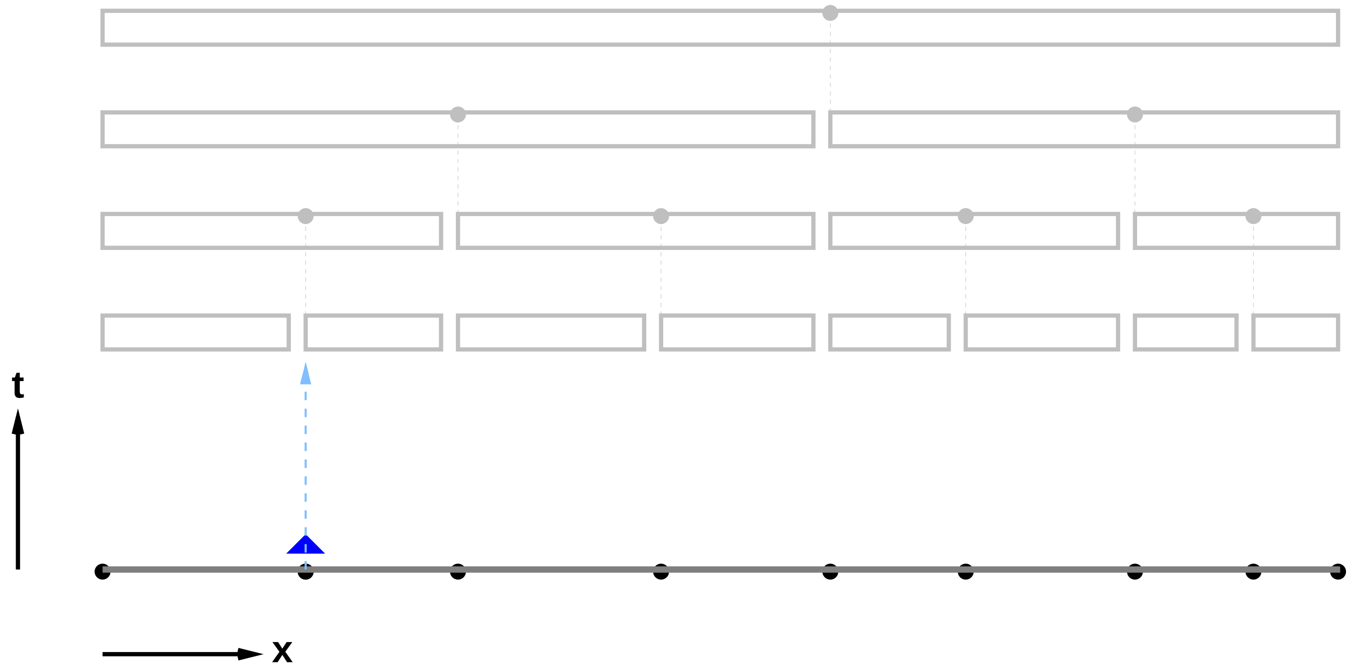


- Non-constant wavespeed means strictest cone constraint can be arbitrarily far away.
- Expensive to examine all cone constraints.

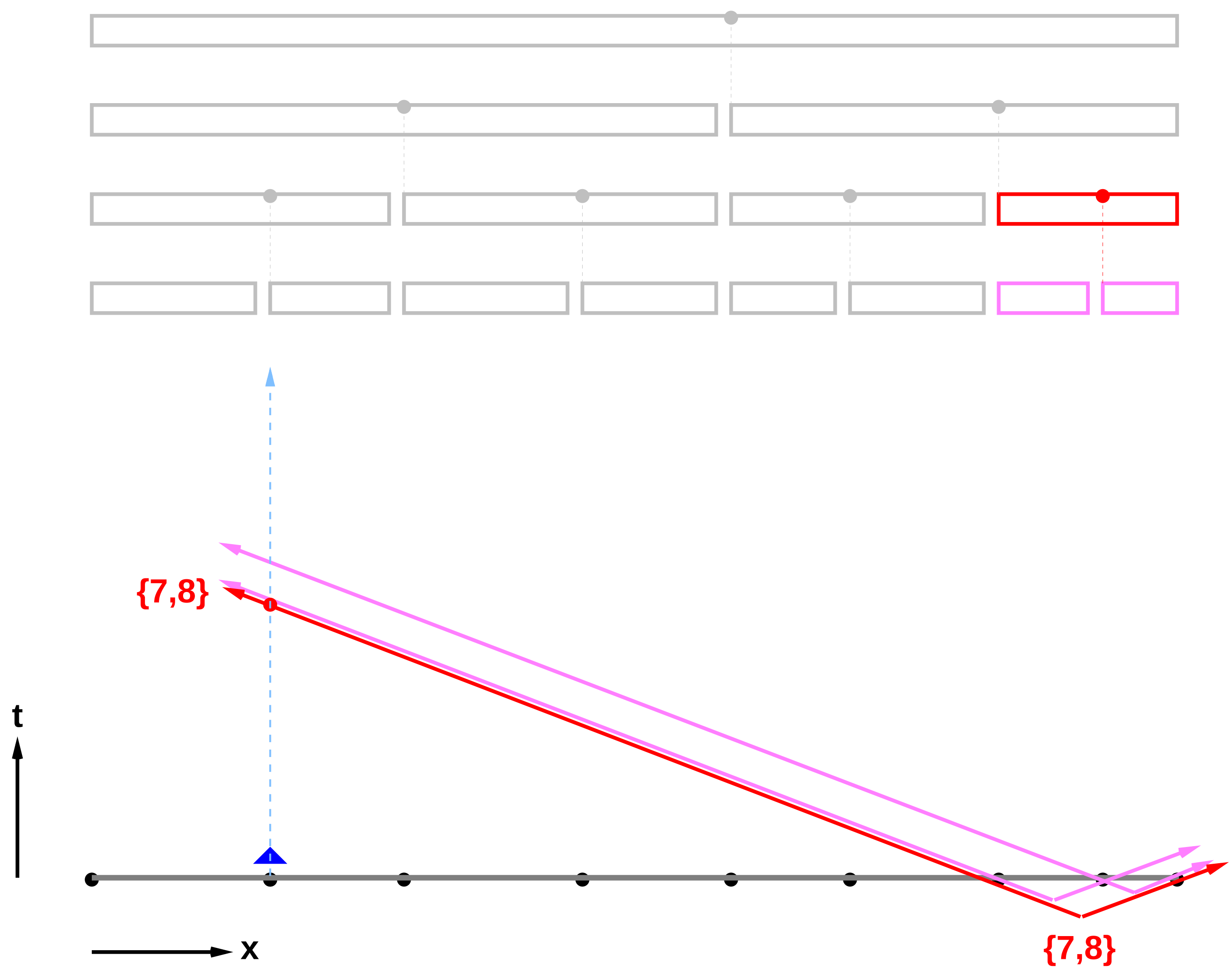
Bounding cone hierarchy

- Adapt standard technique used in computational geometry.
- Solve optimization problem: maximize height of tentpole subject to all cone constraints.
- Group constraints into a hierarchy.
- As expected, only a few constraints in the hierarchy need to be examined on average.

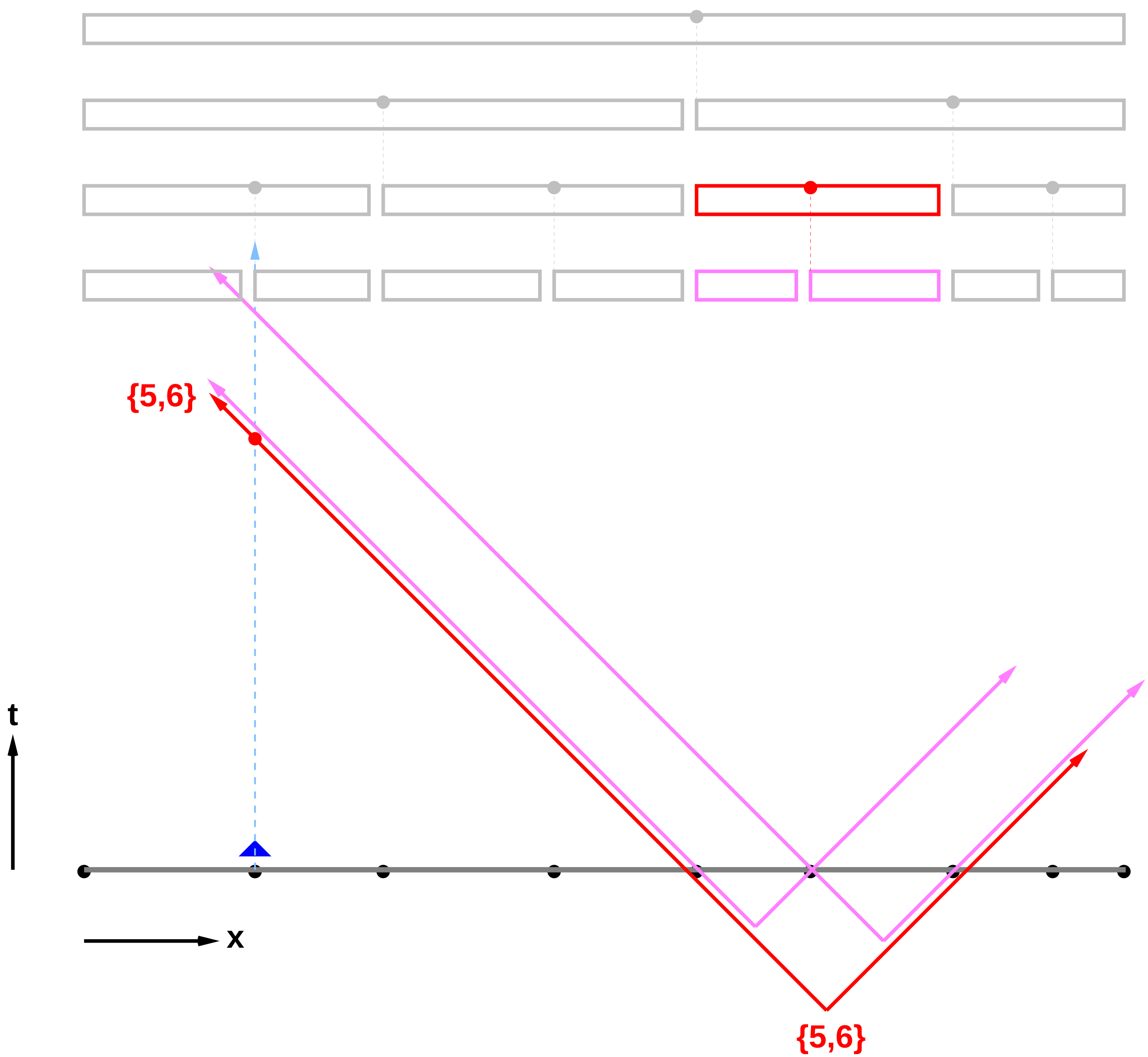
Hierarchical subdivision of domain



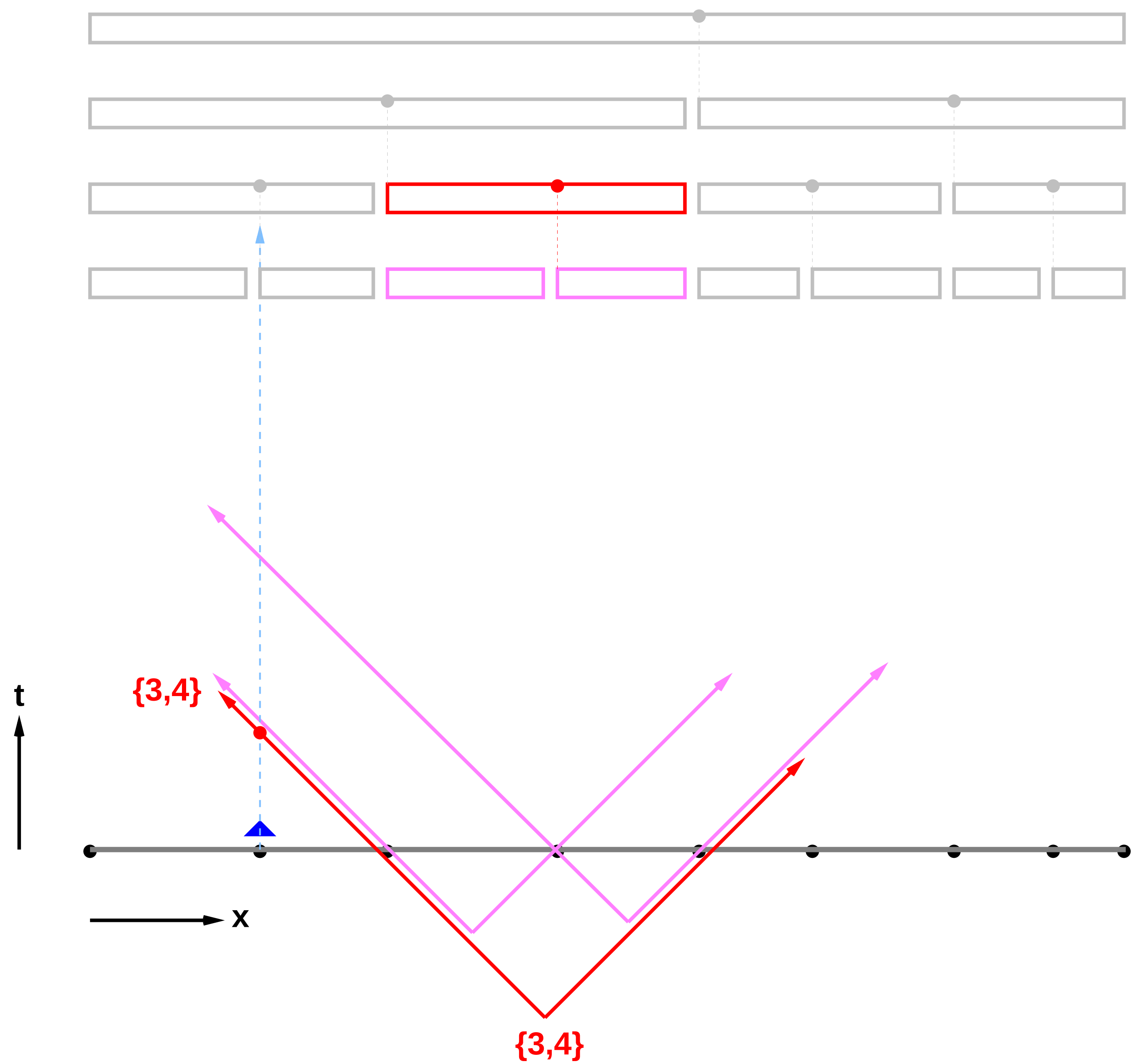
Building the hierarchy



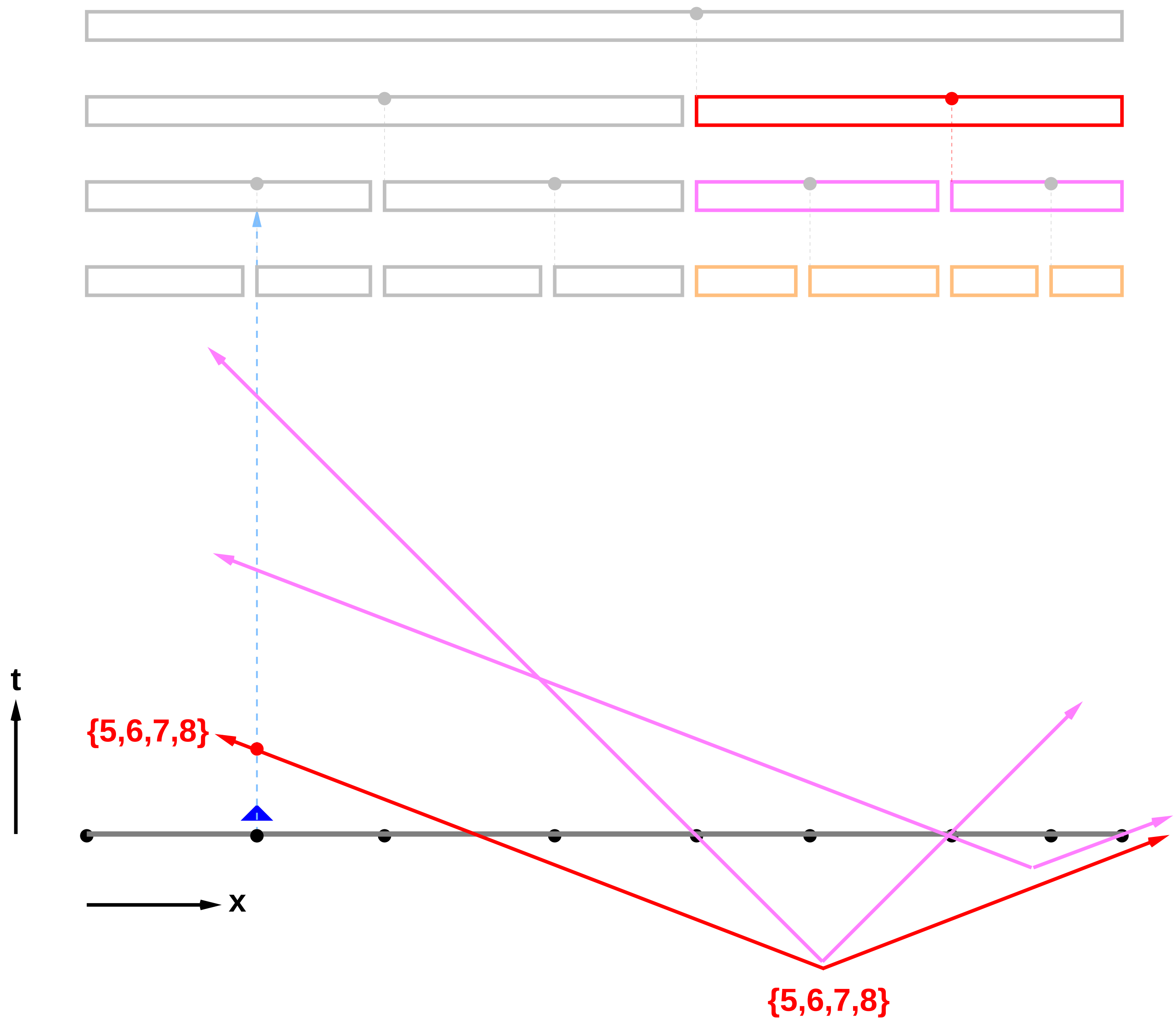
Building the hierarchy (2)



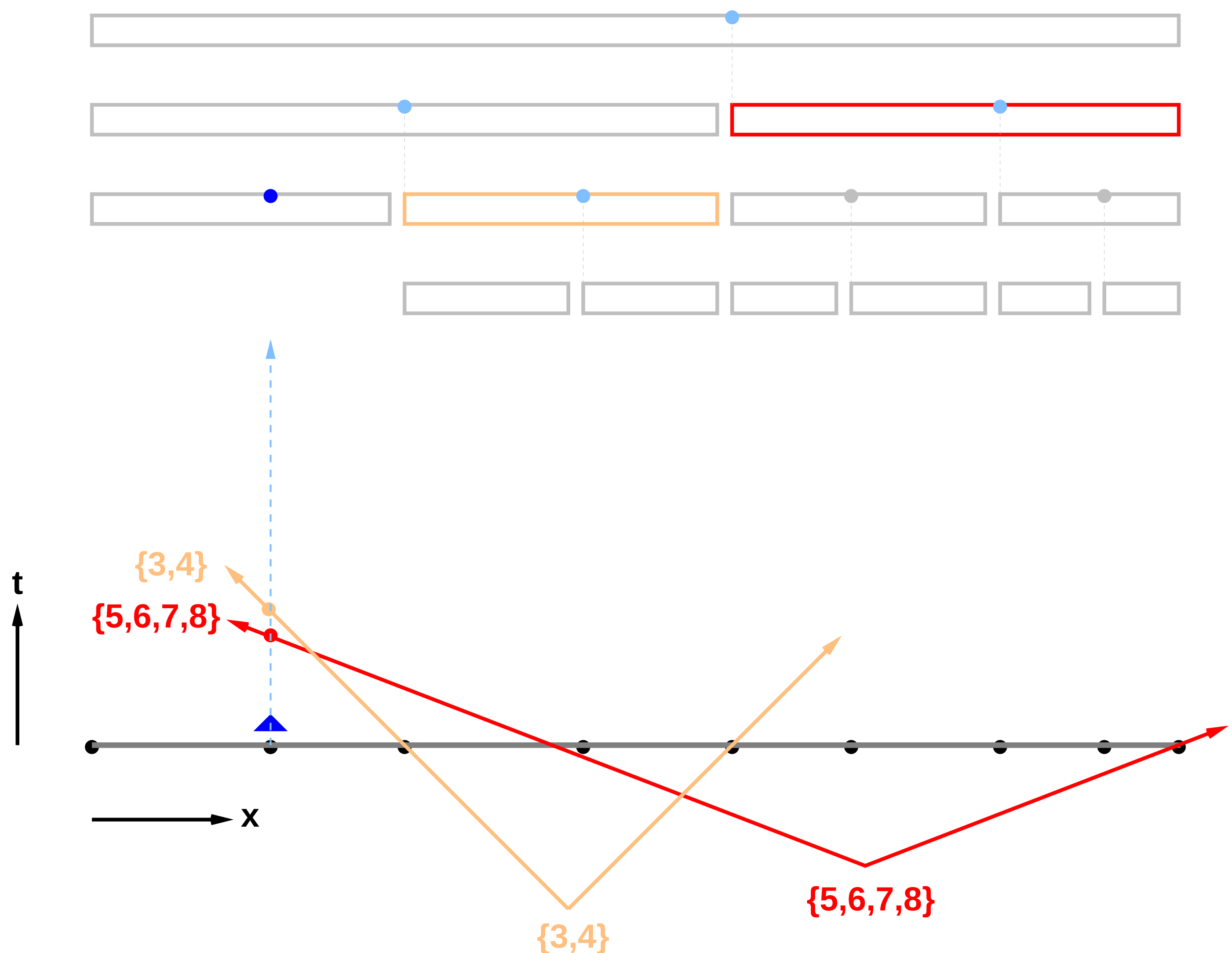
Building the hierarchy (3)



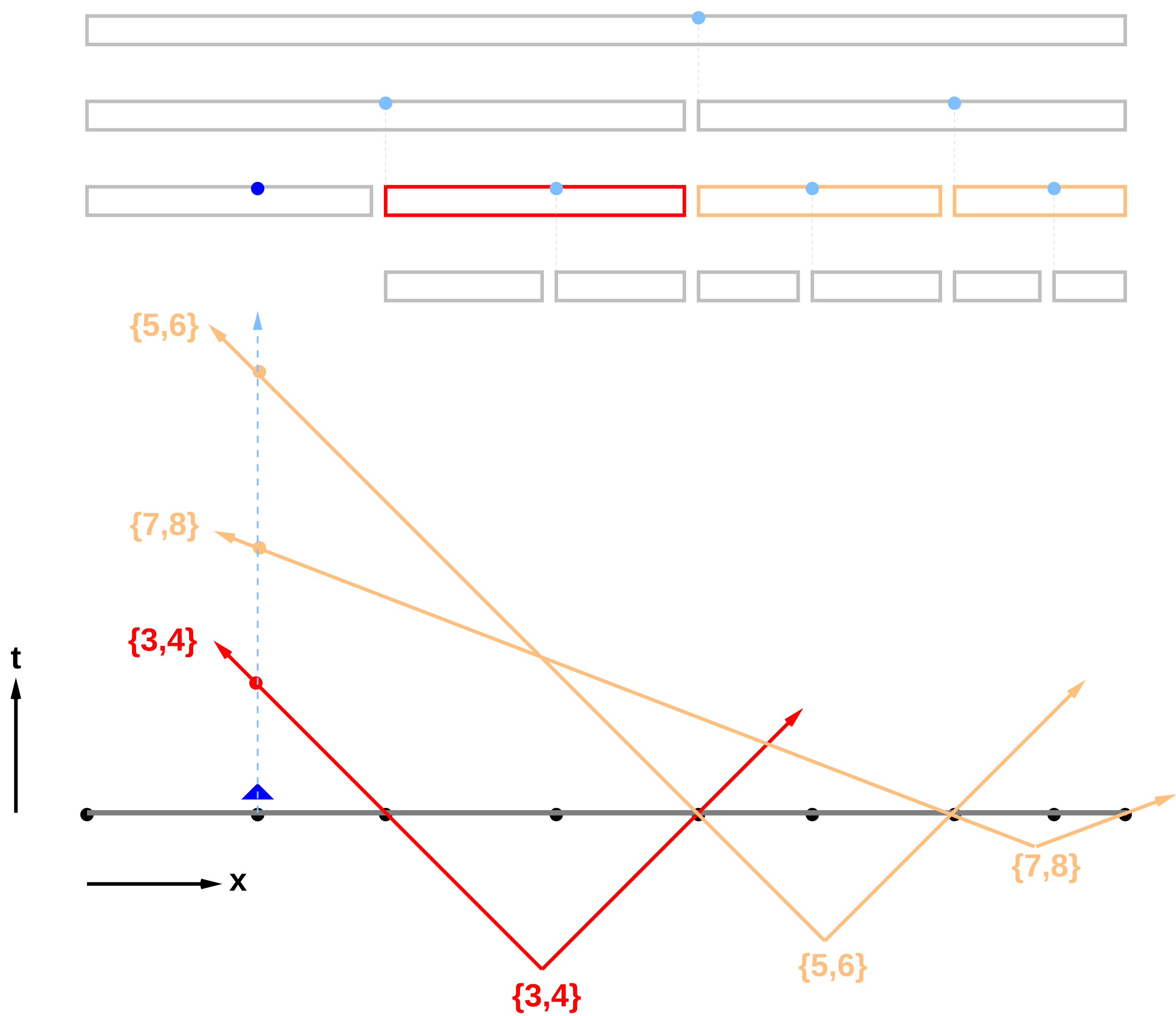
Building the hierarchy (4)



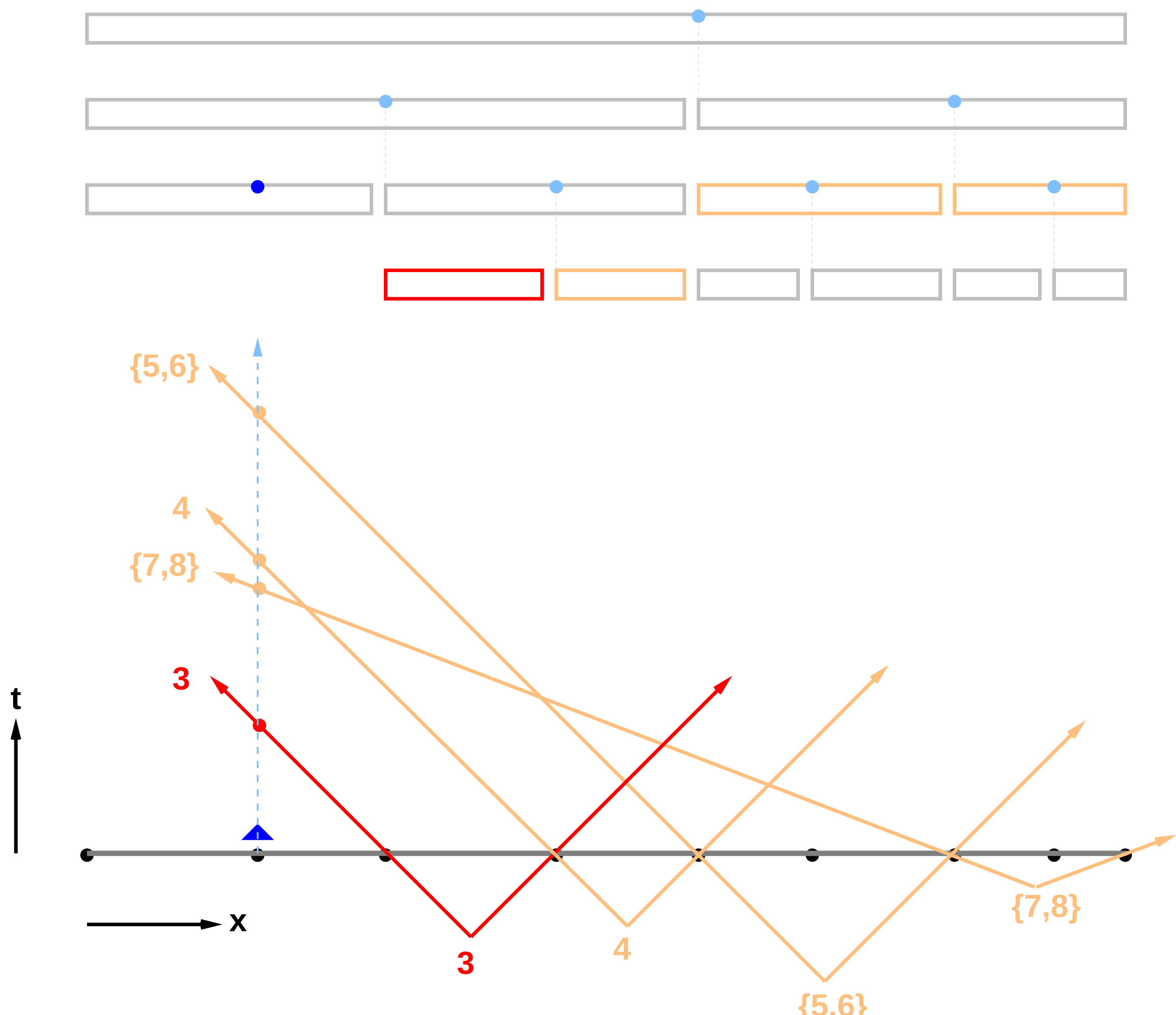
Traversing the hierarchy



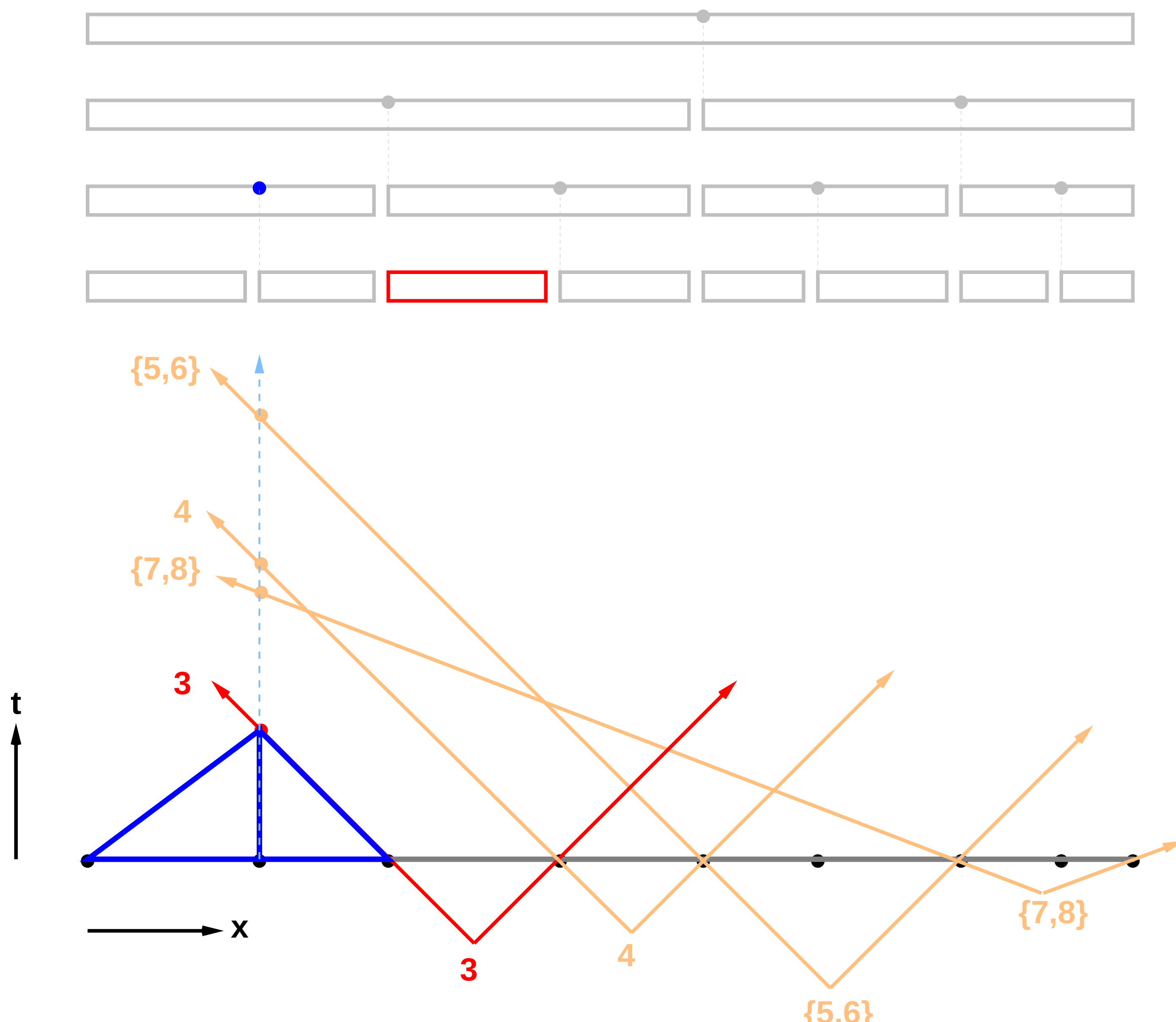
Traversing the hierarchy (2)



Traversing the hierarchy (3)



Traversing the hierarchy (4)



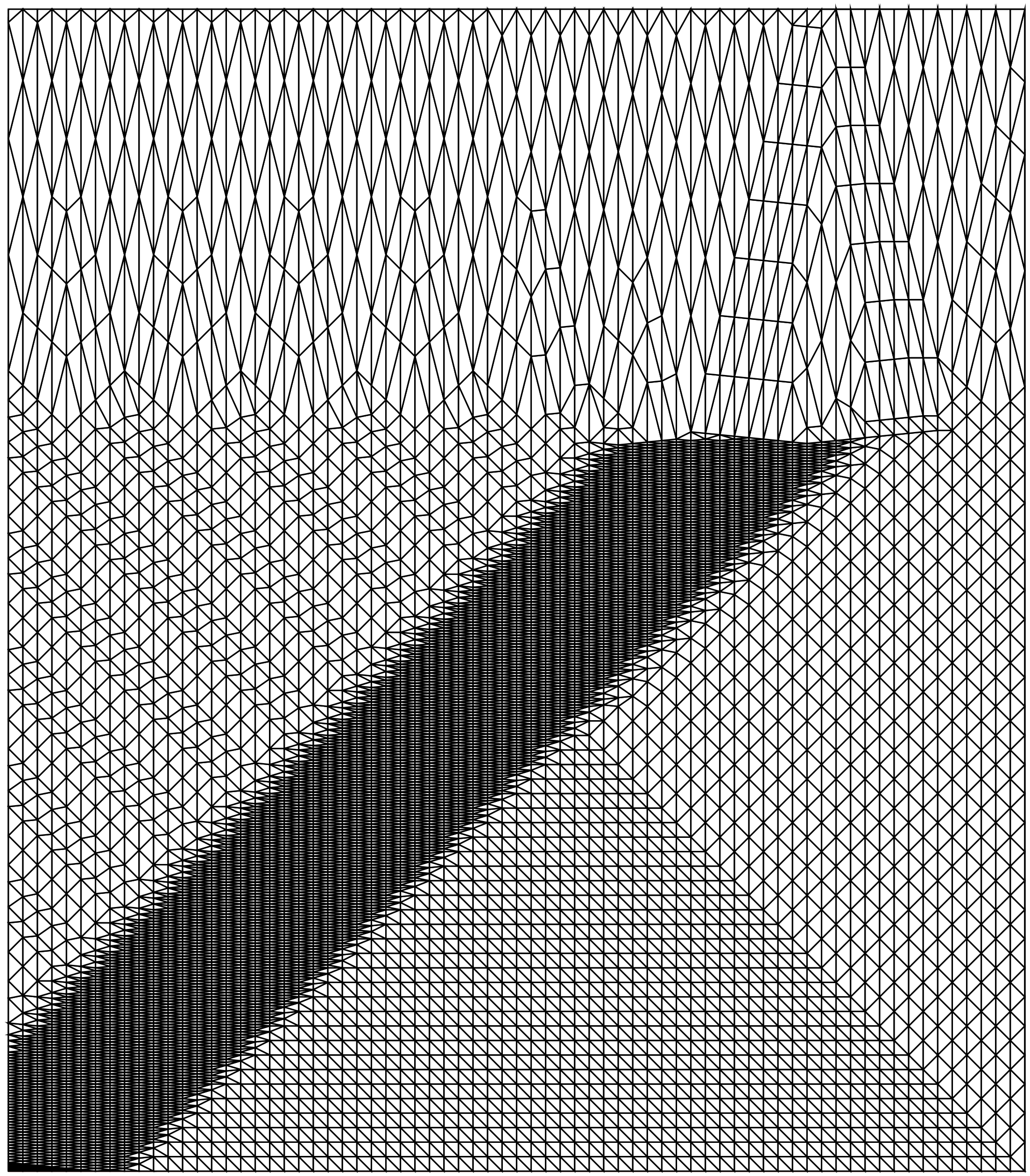
Implementation

We implemented a prototype, for meshing an arbitrary 1D $\times$ time spacetime domain, with the following features:

- Bounding cone hierarchy to efficiently measure nonlocal constraints.
- Adaptive refinement and coarsening.
- Creation of multiple independent patches that can be solved in parallel.

Traversing cone hierarchy verified to be efficient on average.

Example



Work in progress and some open problems

- Implementation for 2D $\times$ time domains.
- Parallel mesh generation and solution on clusters.
- Heuristics for maximizing amount of progress in each iteration.
- Quality criteria for a “good” spacetime mesh.

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Reza Abedi, Jonathan Booth, Shuo-Heng Chung
Yong Fan, Michael Garland

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