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Pants Decomposition of the Punctured Plane

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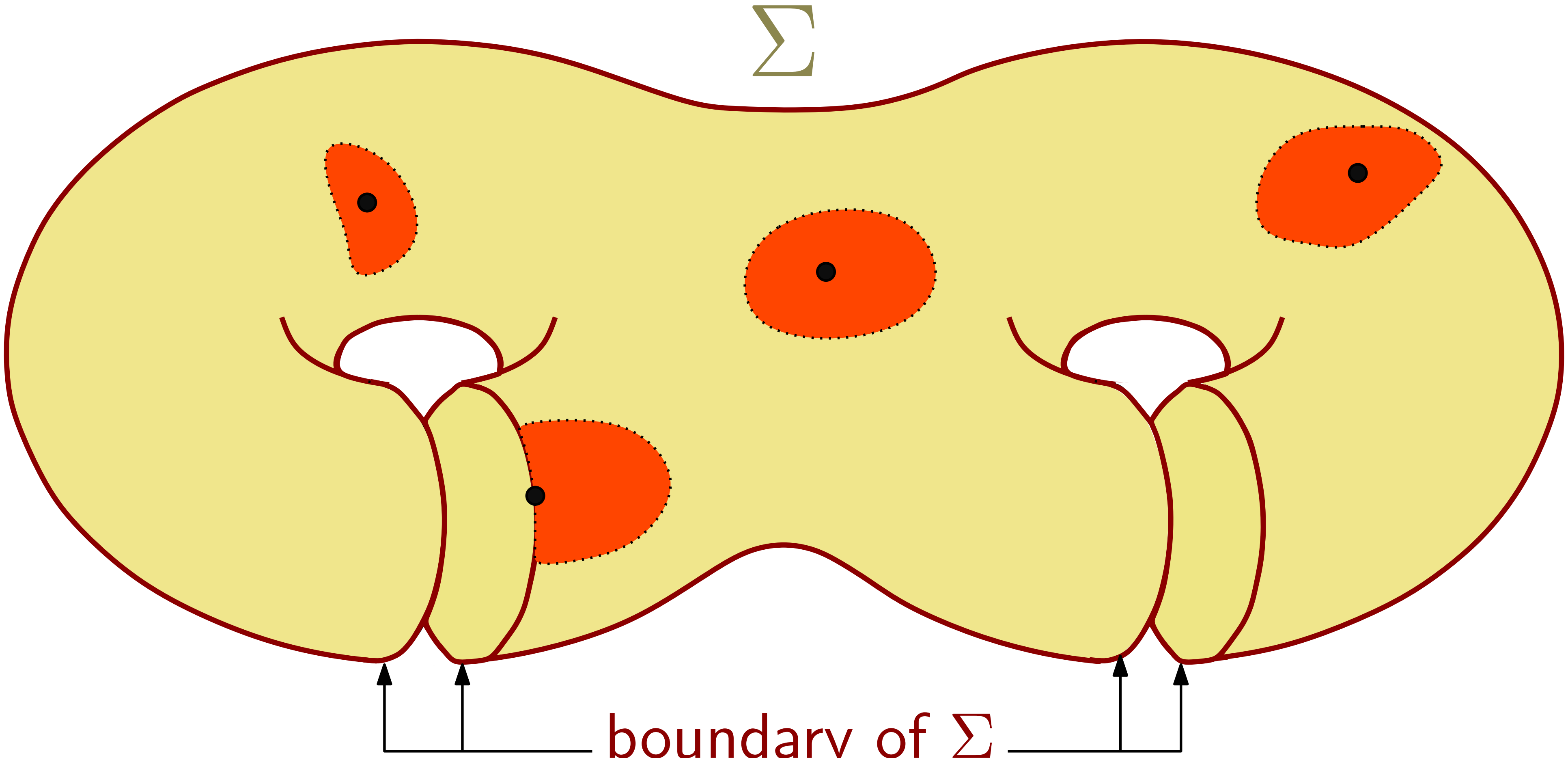
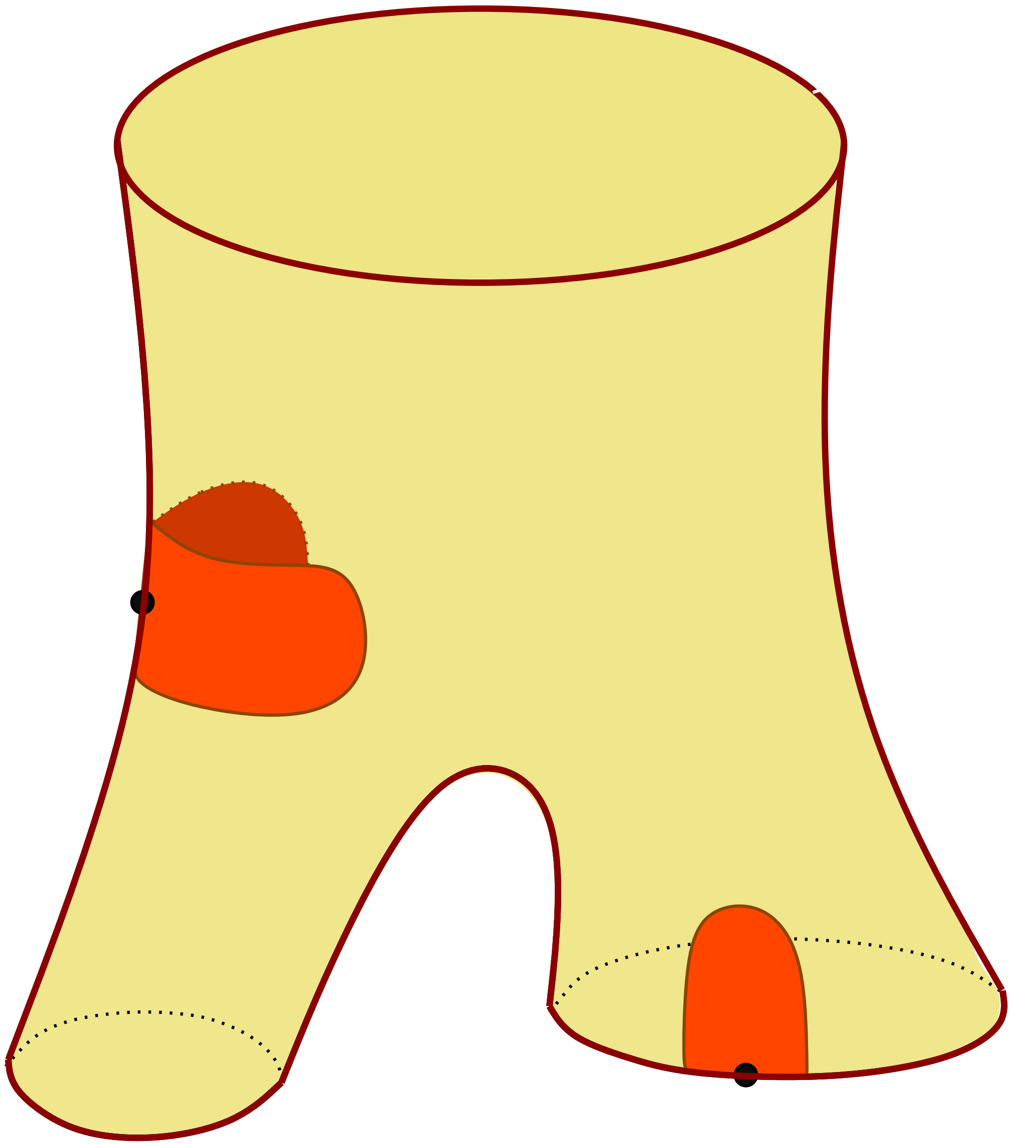
Surface $\equiv$ 2-manifold



Pant $\equiv$ a sphere with 3 holes



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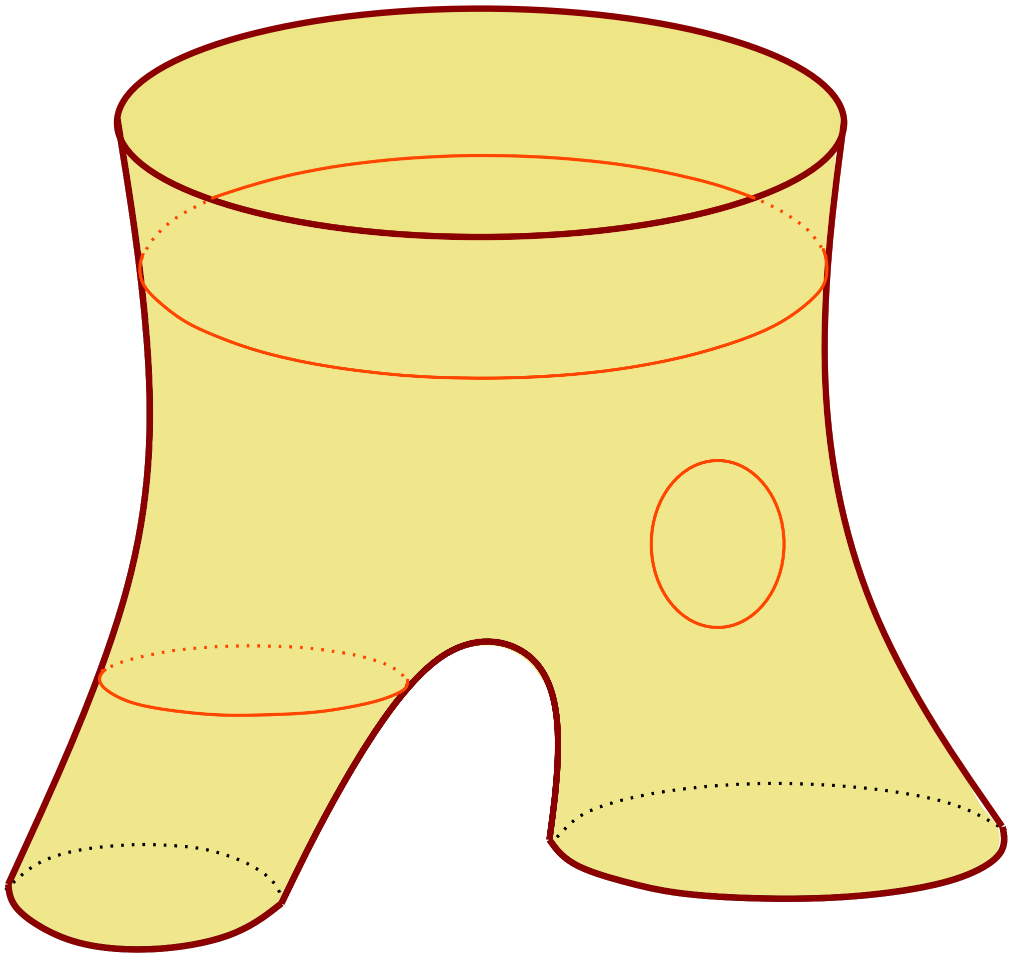


Every simple cycle on a pant is contractible to a point or to a boundary

Pants decomposition

Definition: A set of simple cycles that decompose a surface into disjoint pants

To understand the topology of the surface and to compute its various properties

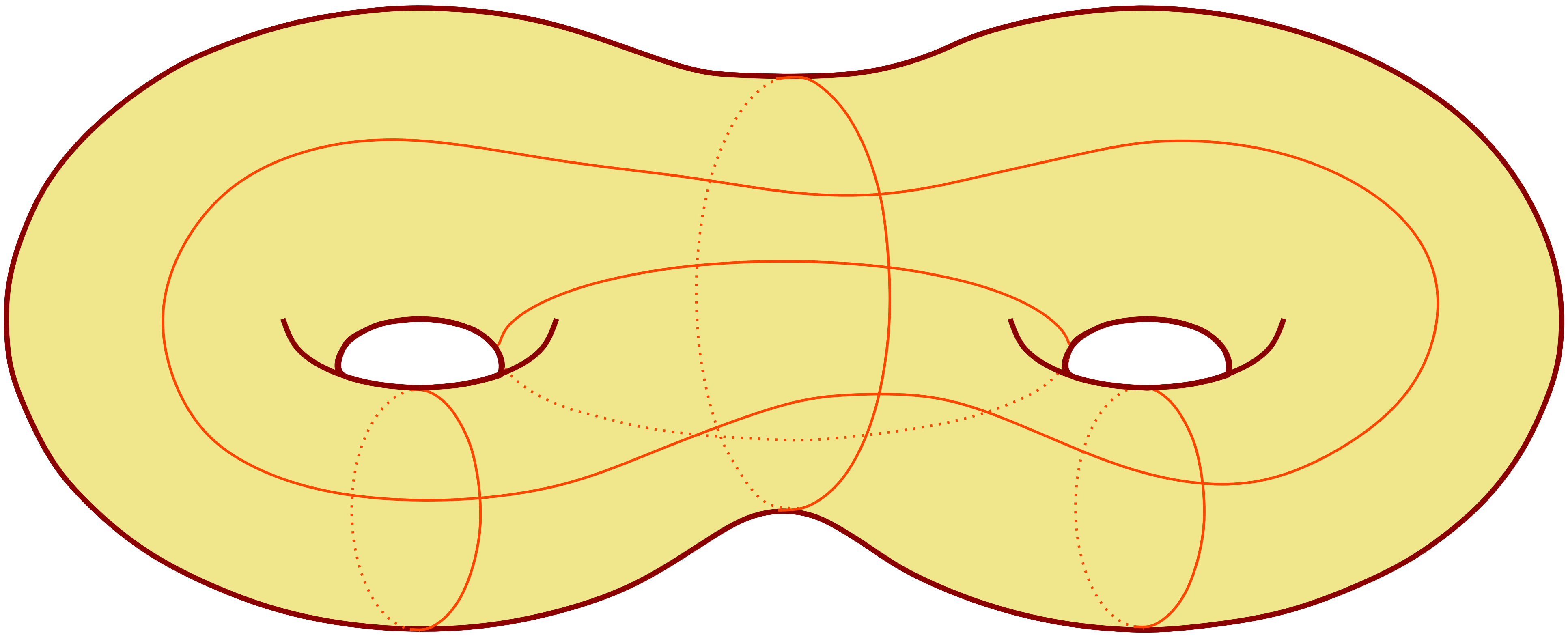
Every compact orientable surface has at least one pants decomposition*

** except sphere, cylinder, disk, or torus*

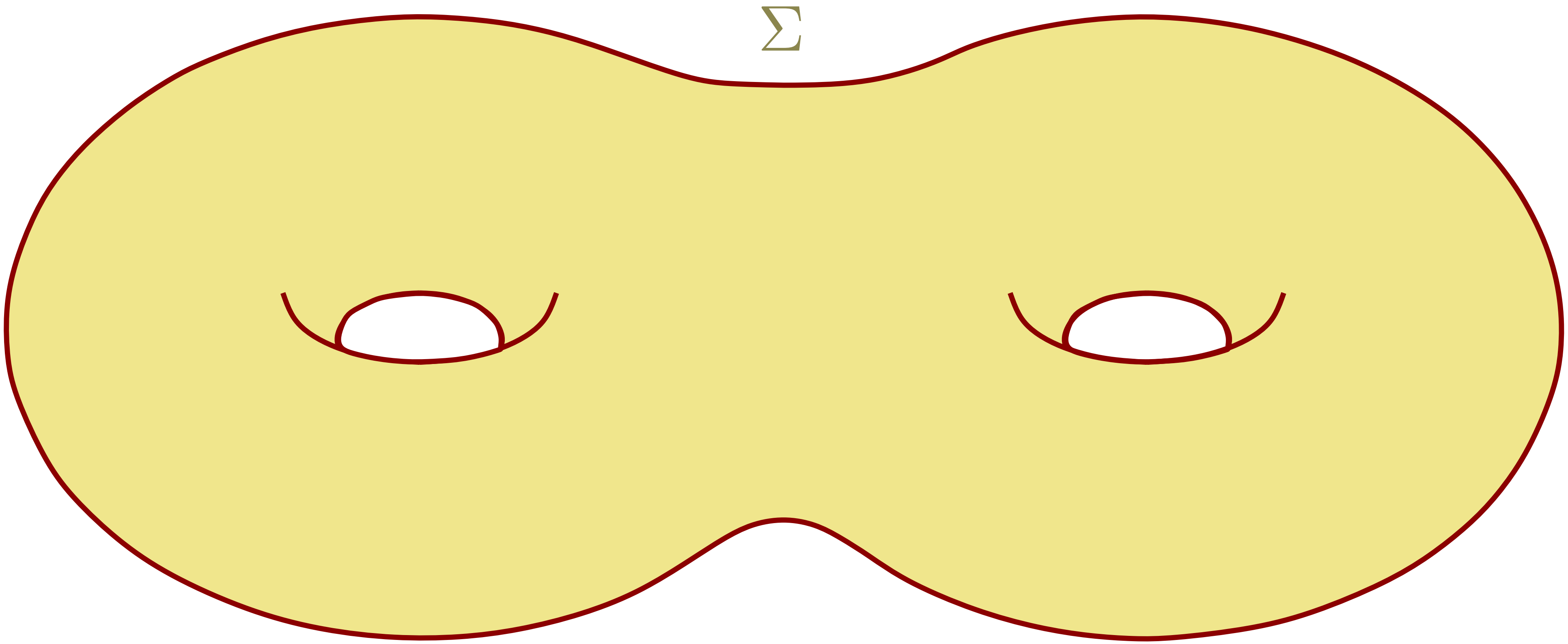
A surface can have many pants decompositions

Essential cycle

Simple cycle not contractible to a point or to a boundary



Example: decomposing into pants



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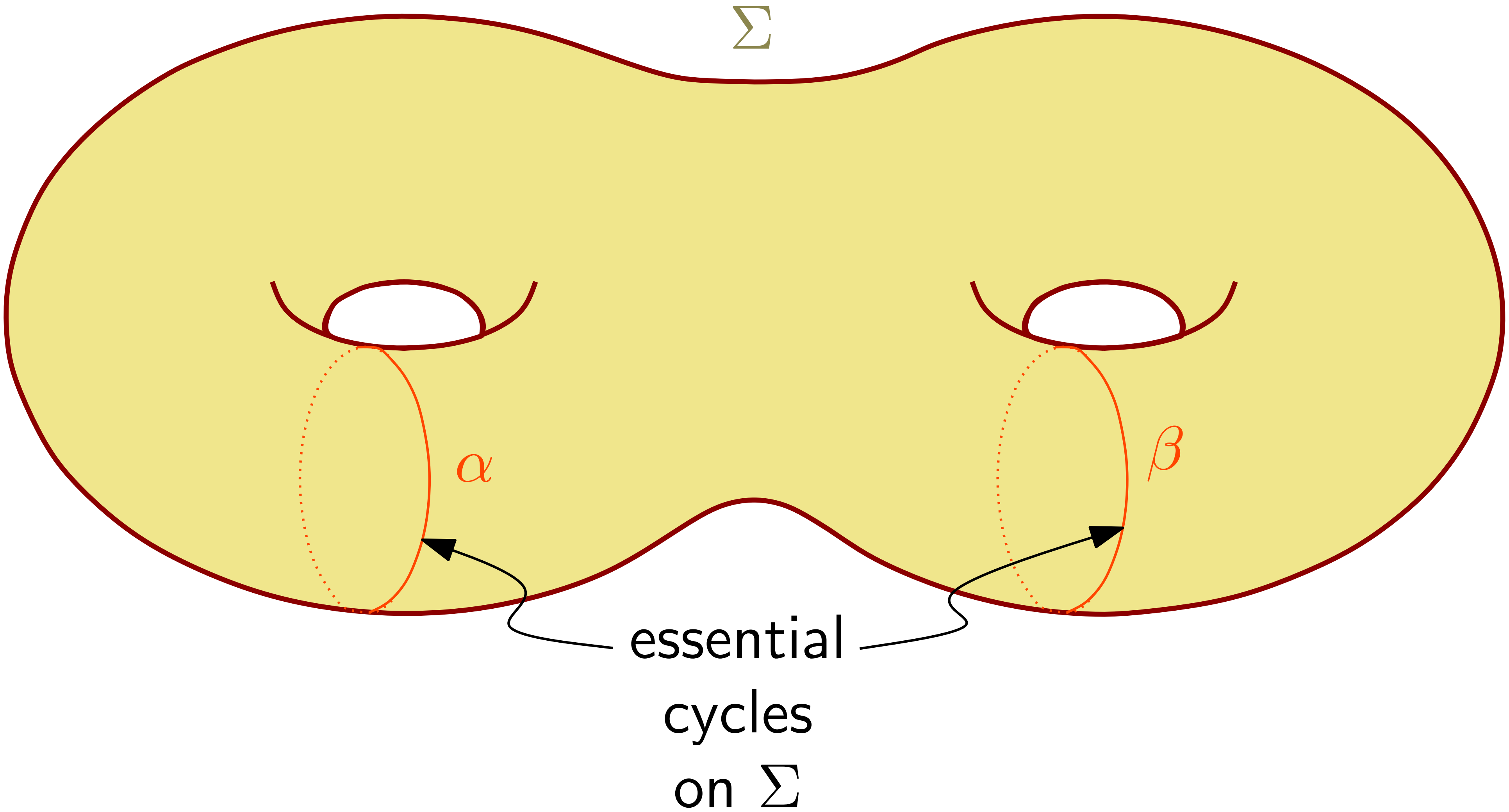

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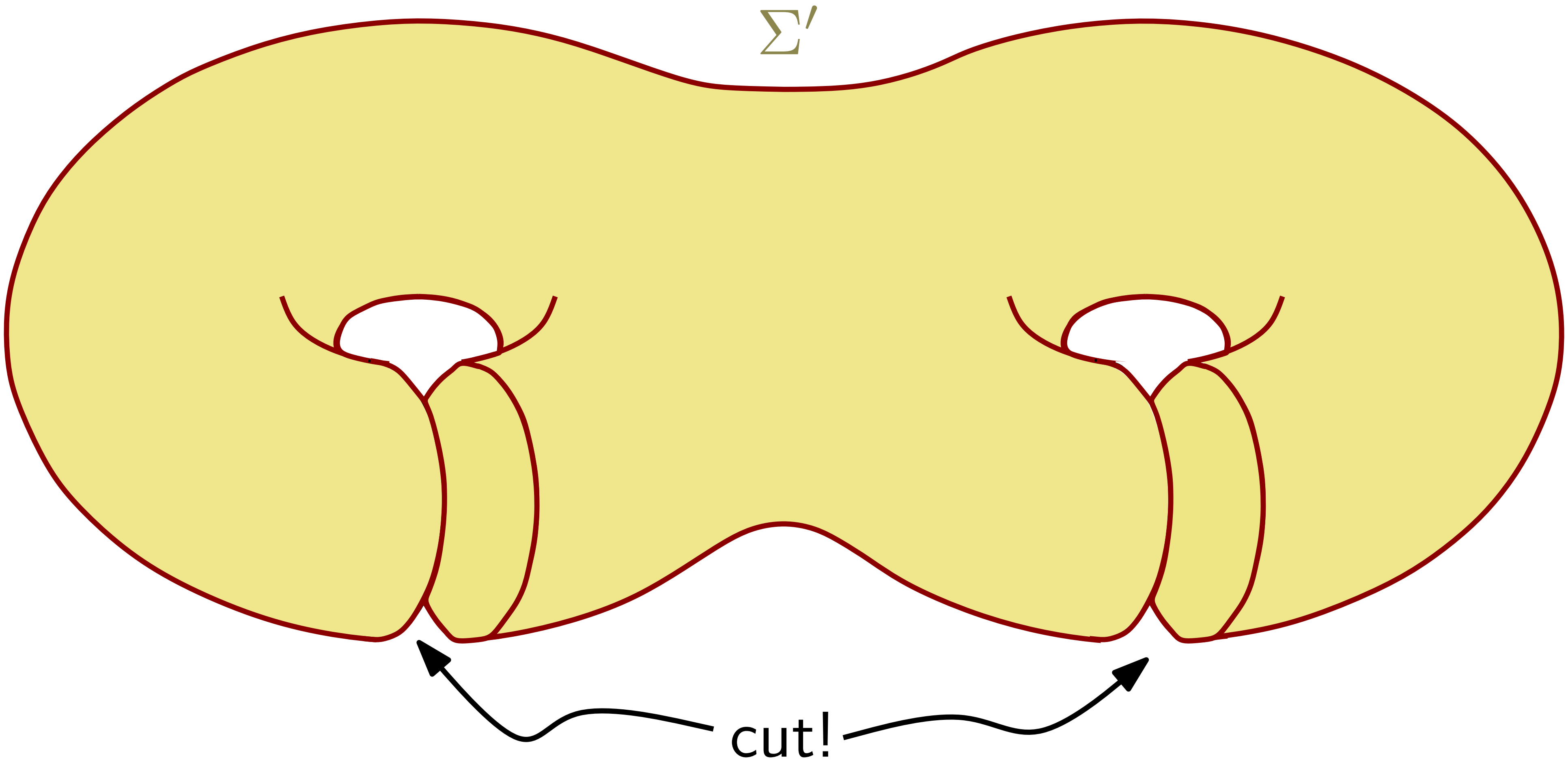

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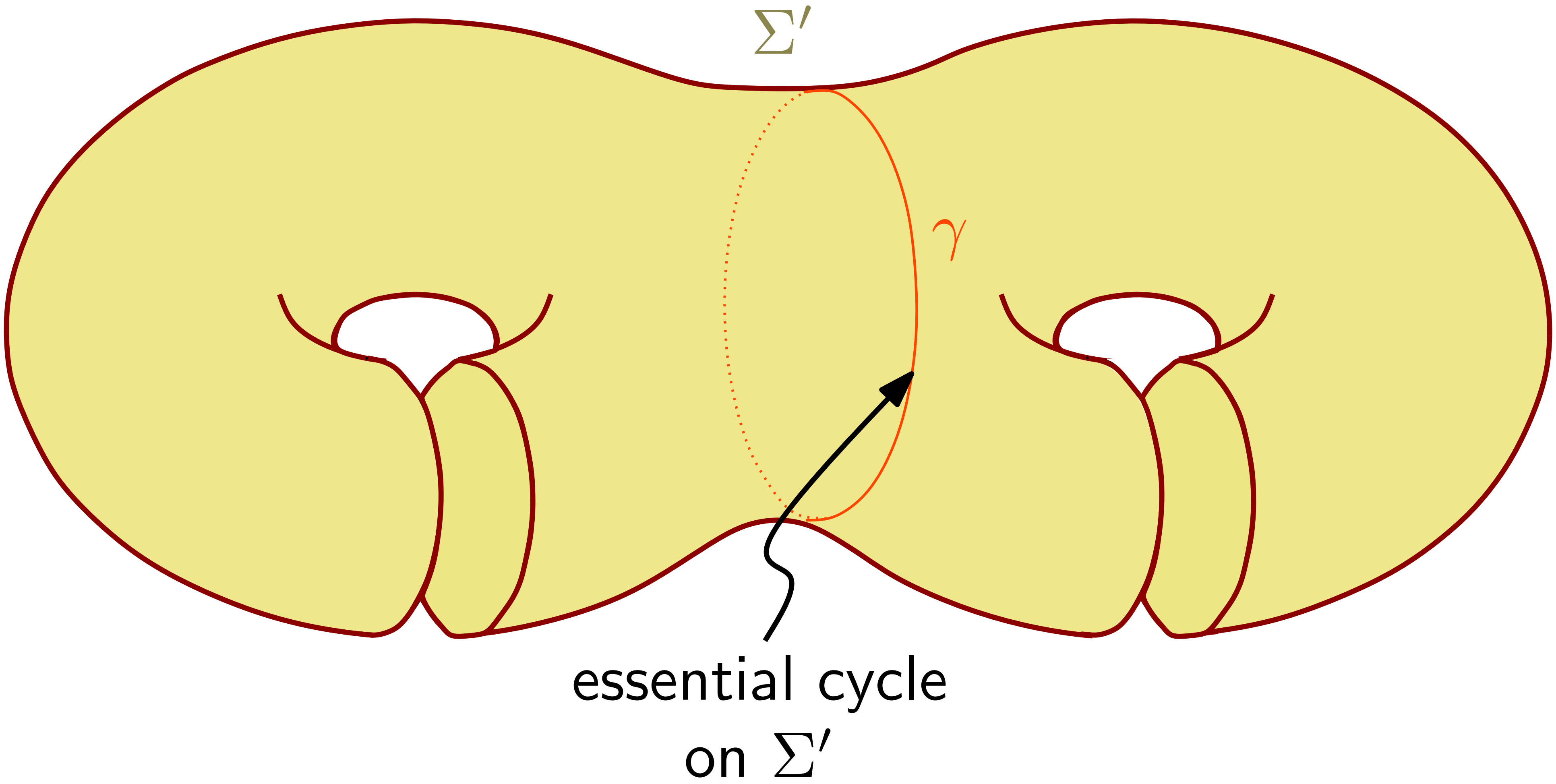

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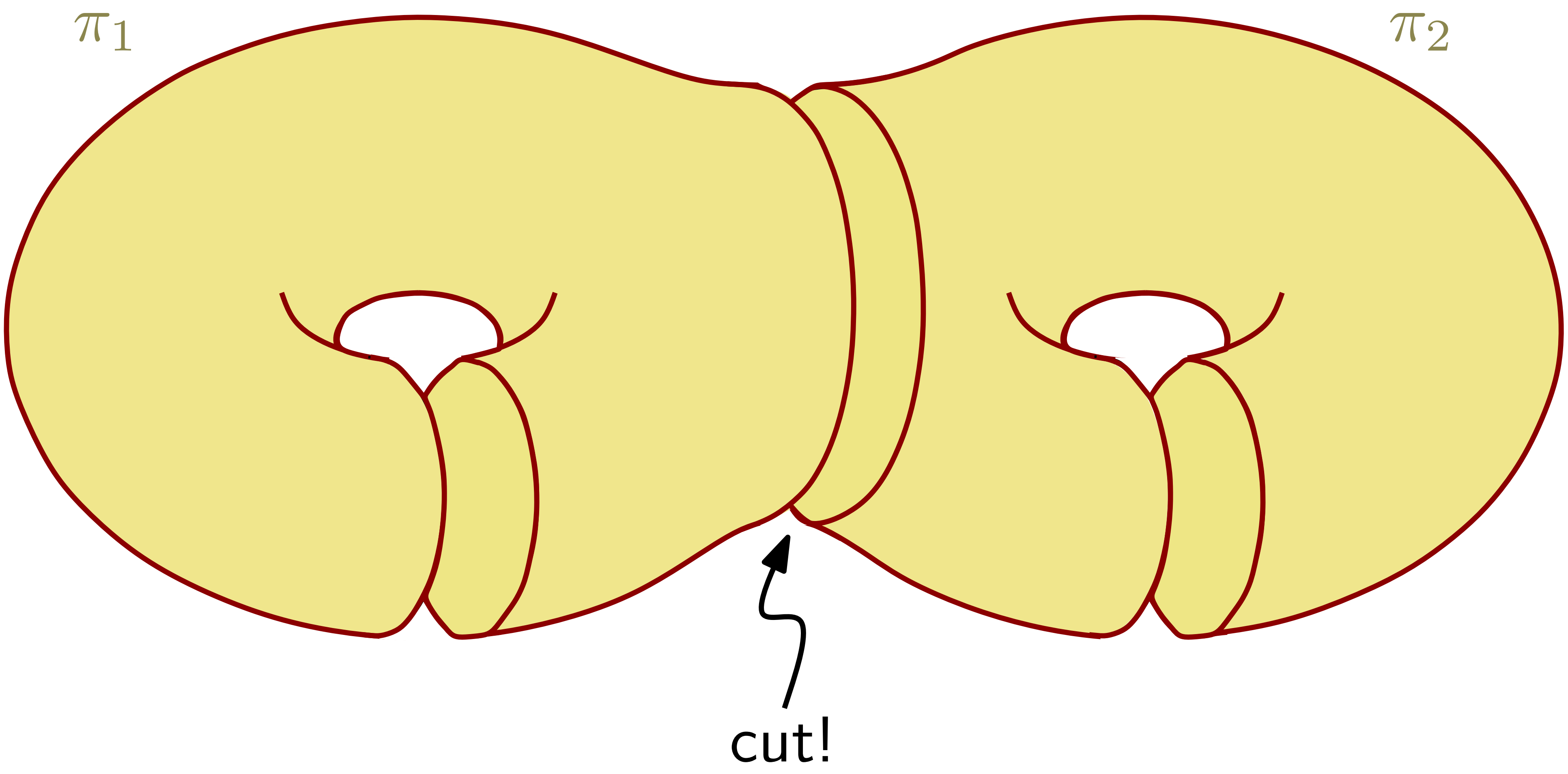

The big open problem

Computing an exact or approximate *shortest* pants decomposition of a general combinatorial surface

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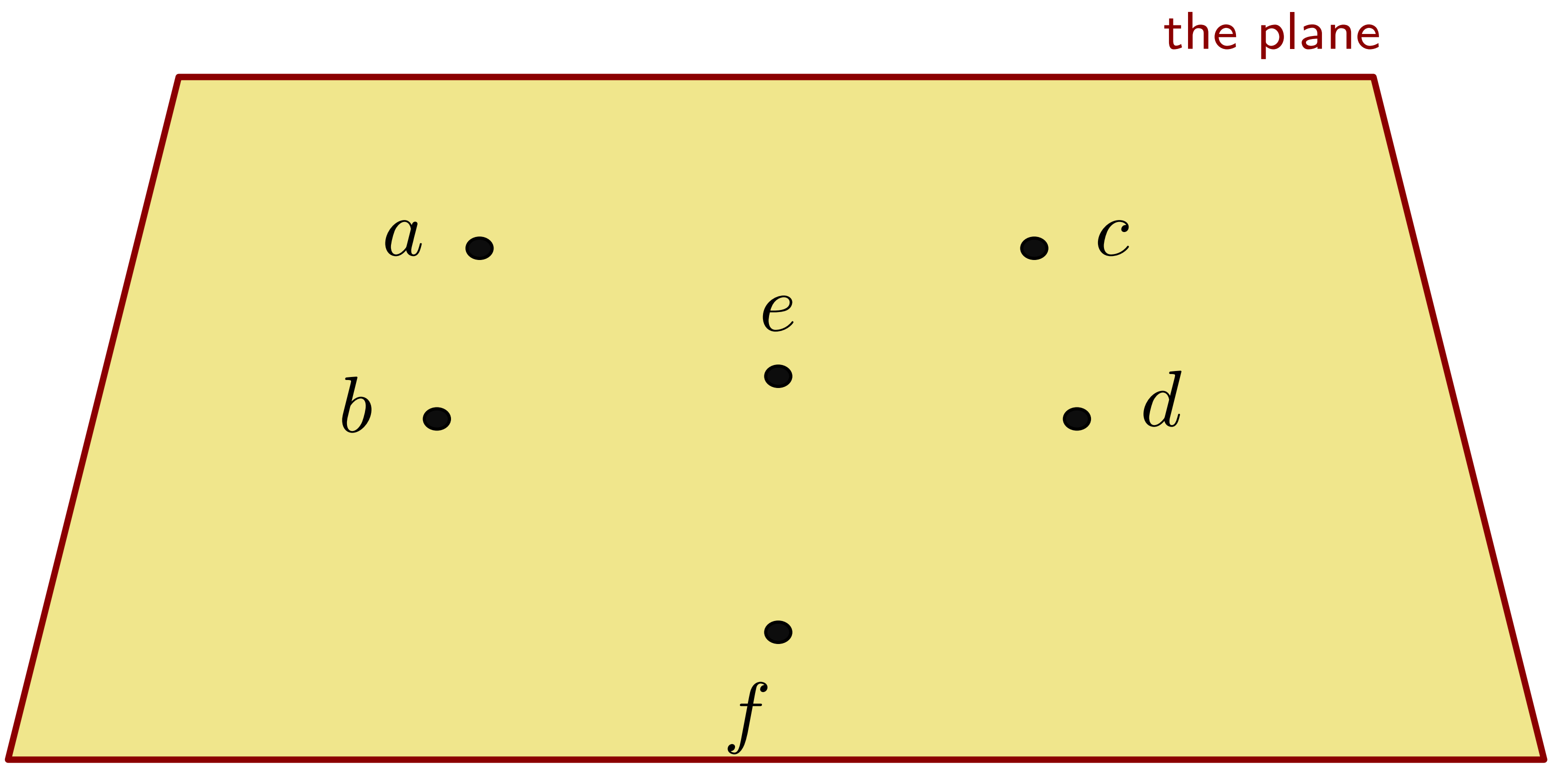
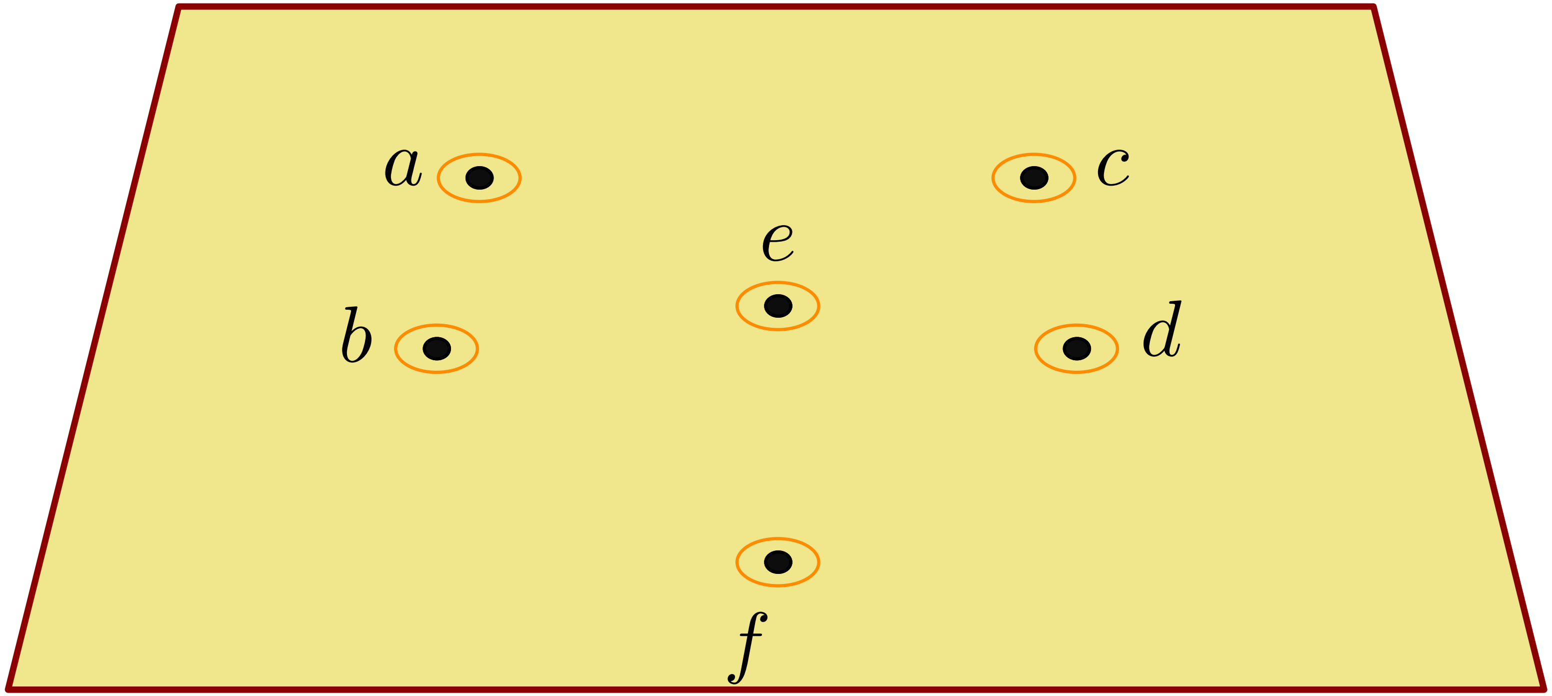
Computing an exact or approximate shortest pants decomposition of a general combinatorial surface

We consider a variant in the Euclidean plane . . .

Punctured plane

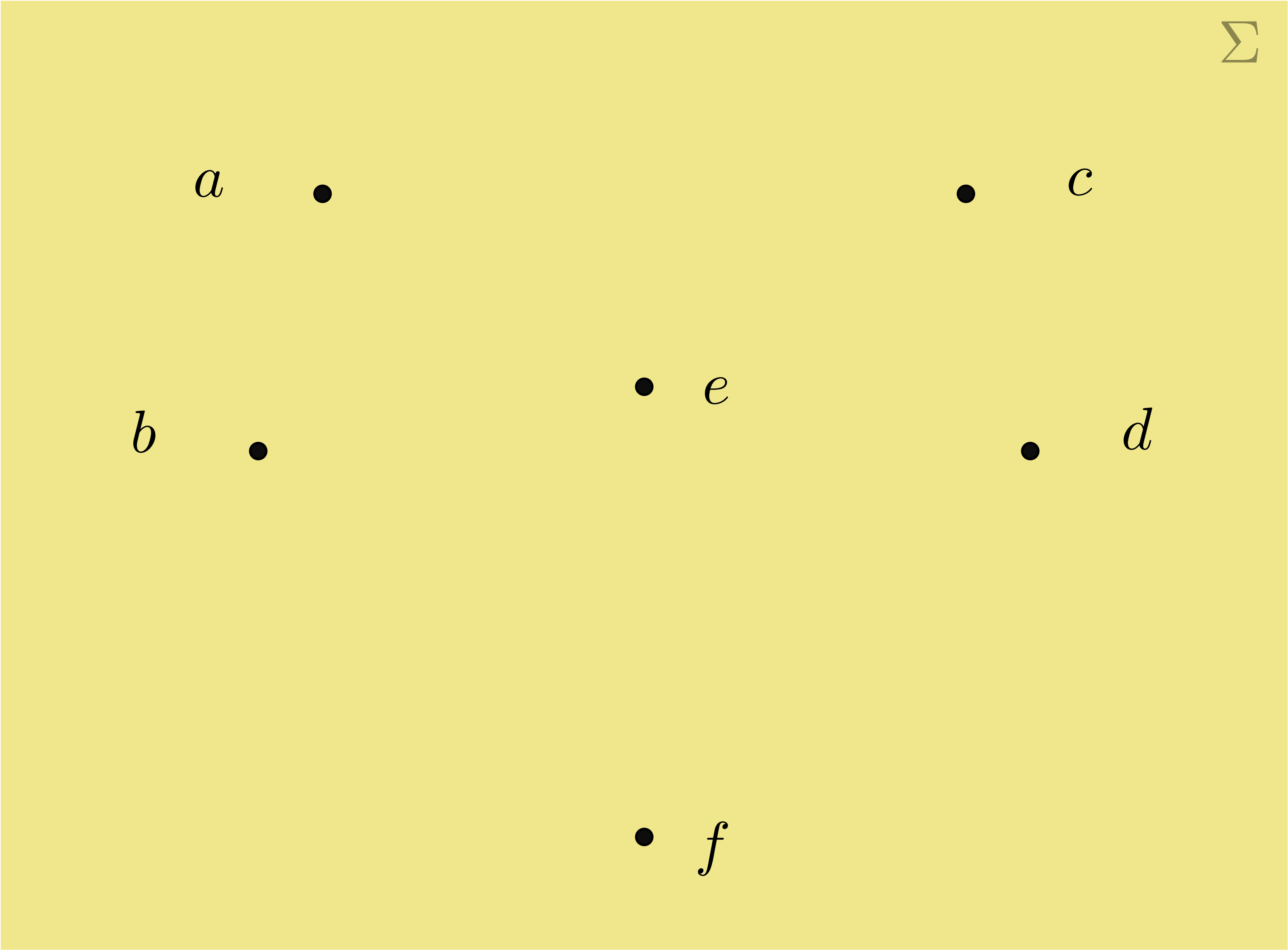


Punctured plane



Surface Σ is the plane minus n points

Decomposing the punctured plane



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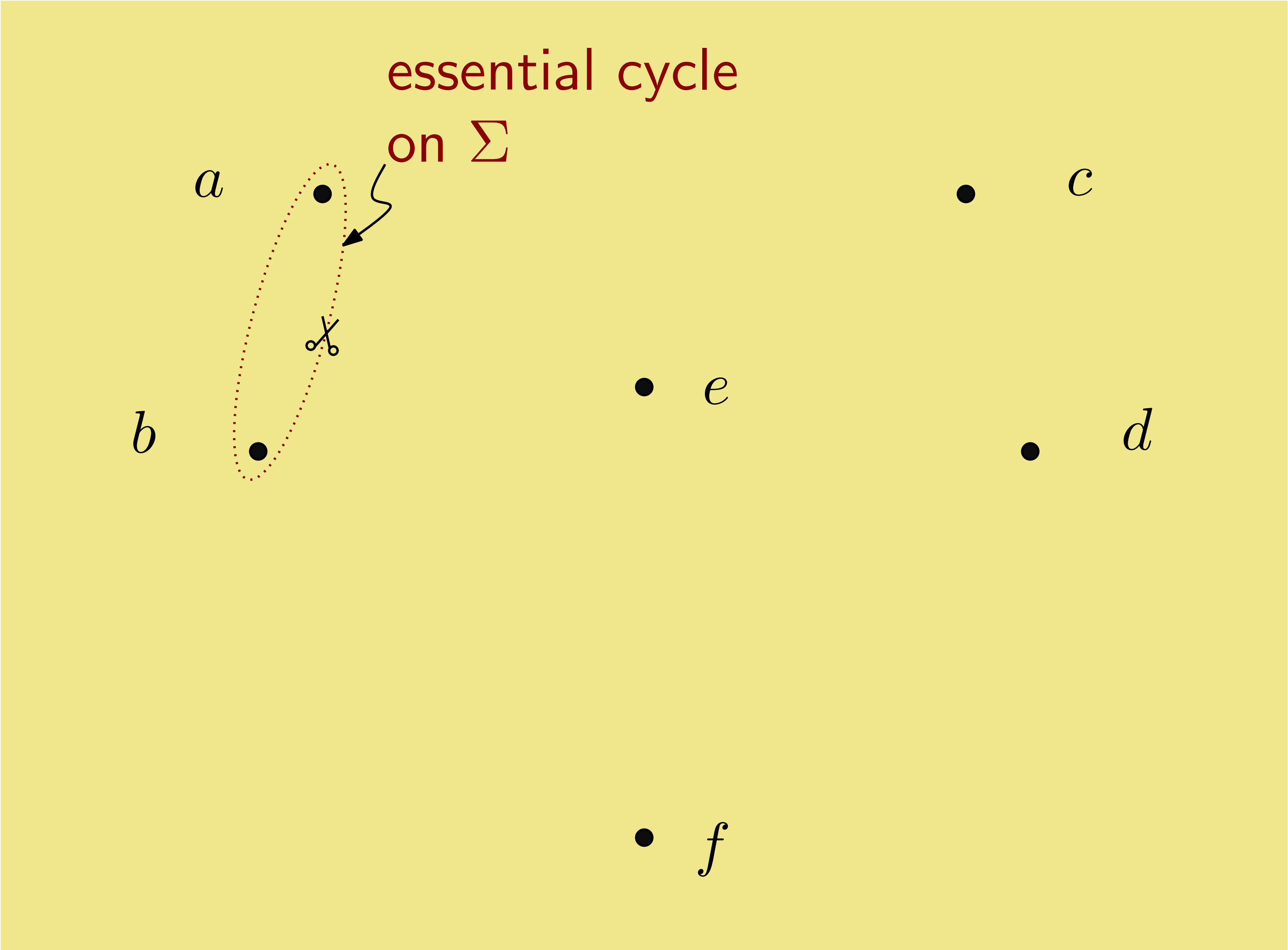

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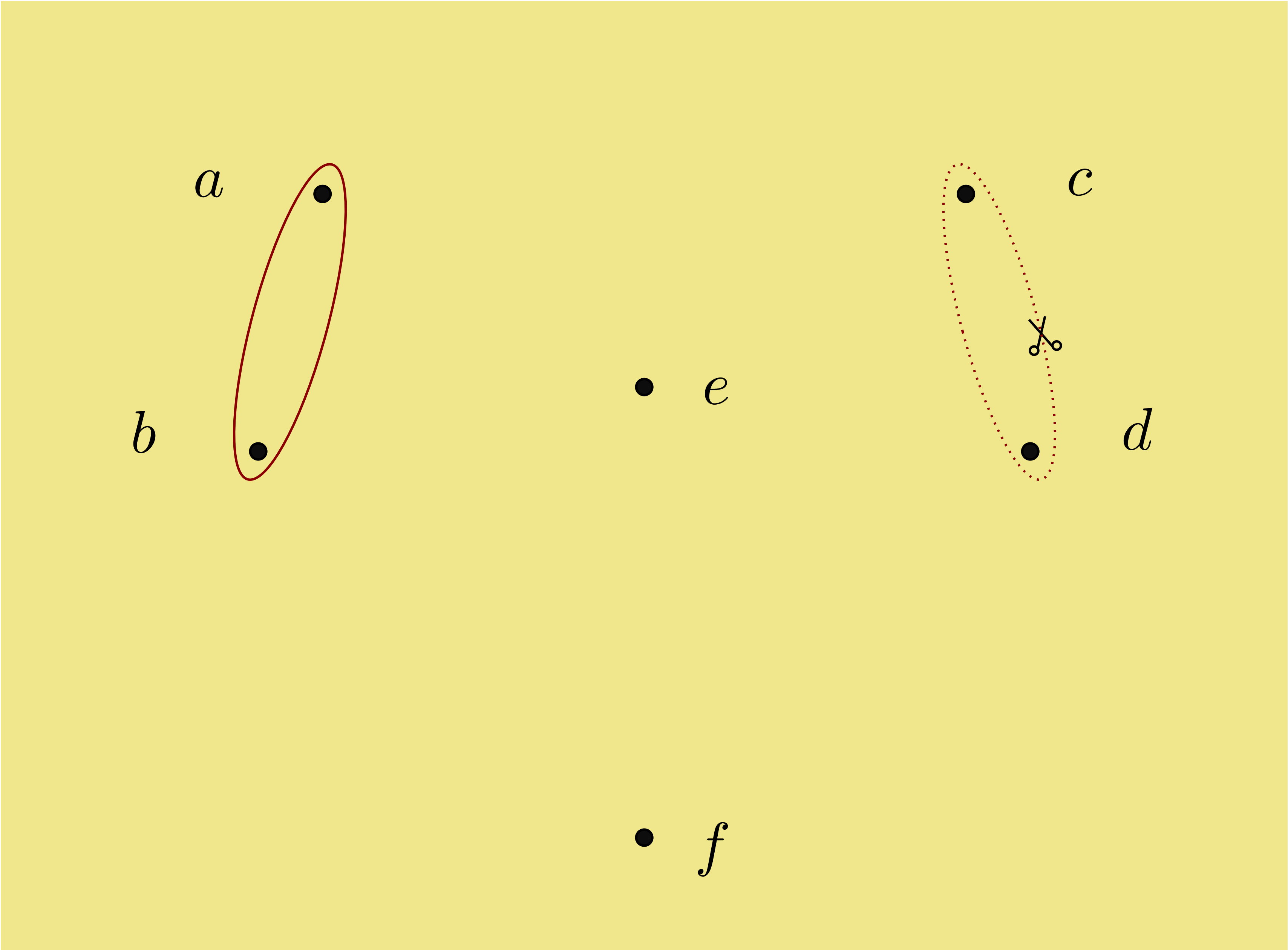

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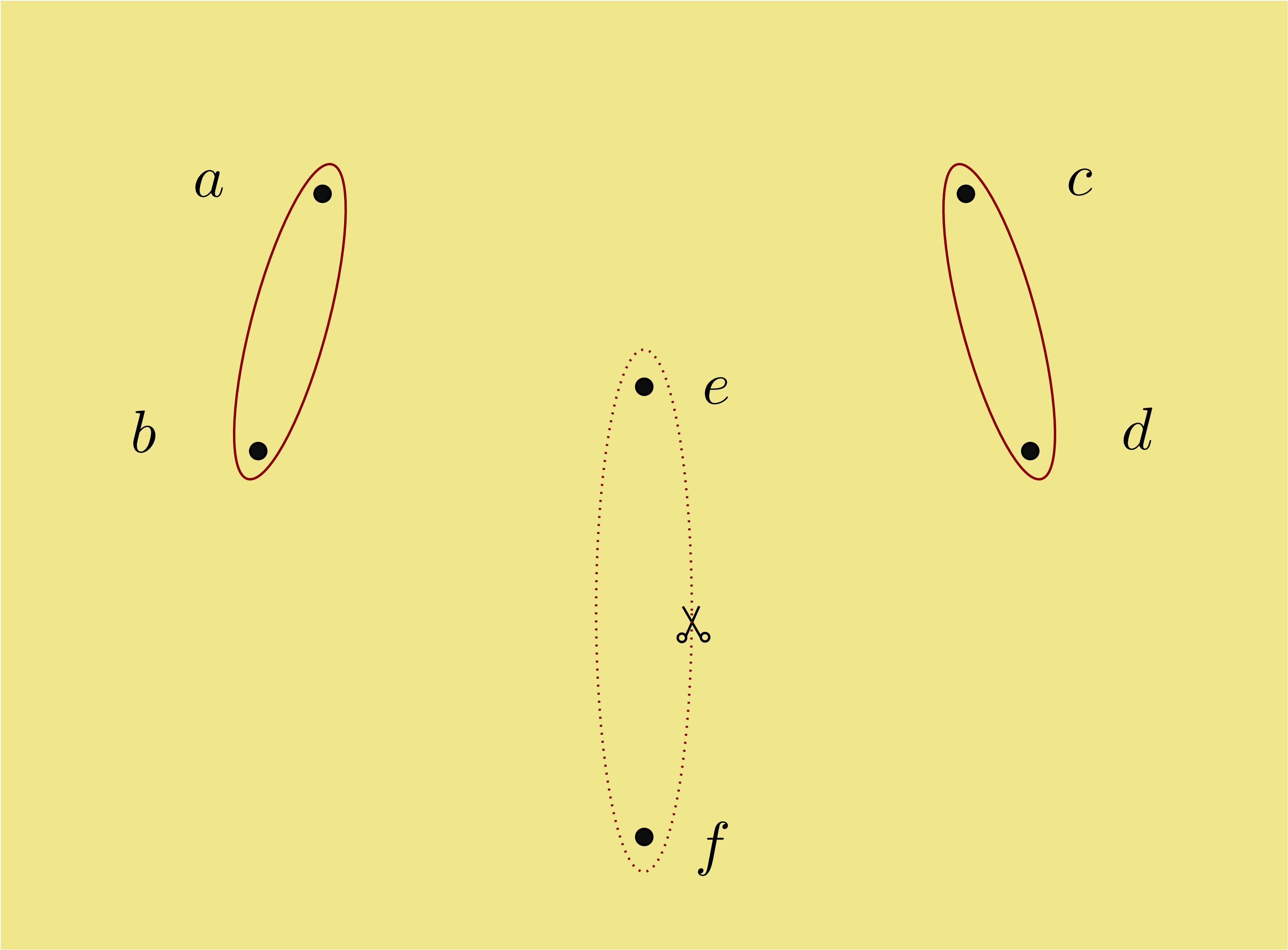

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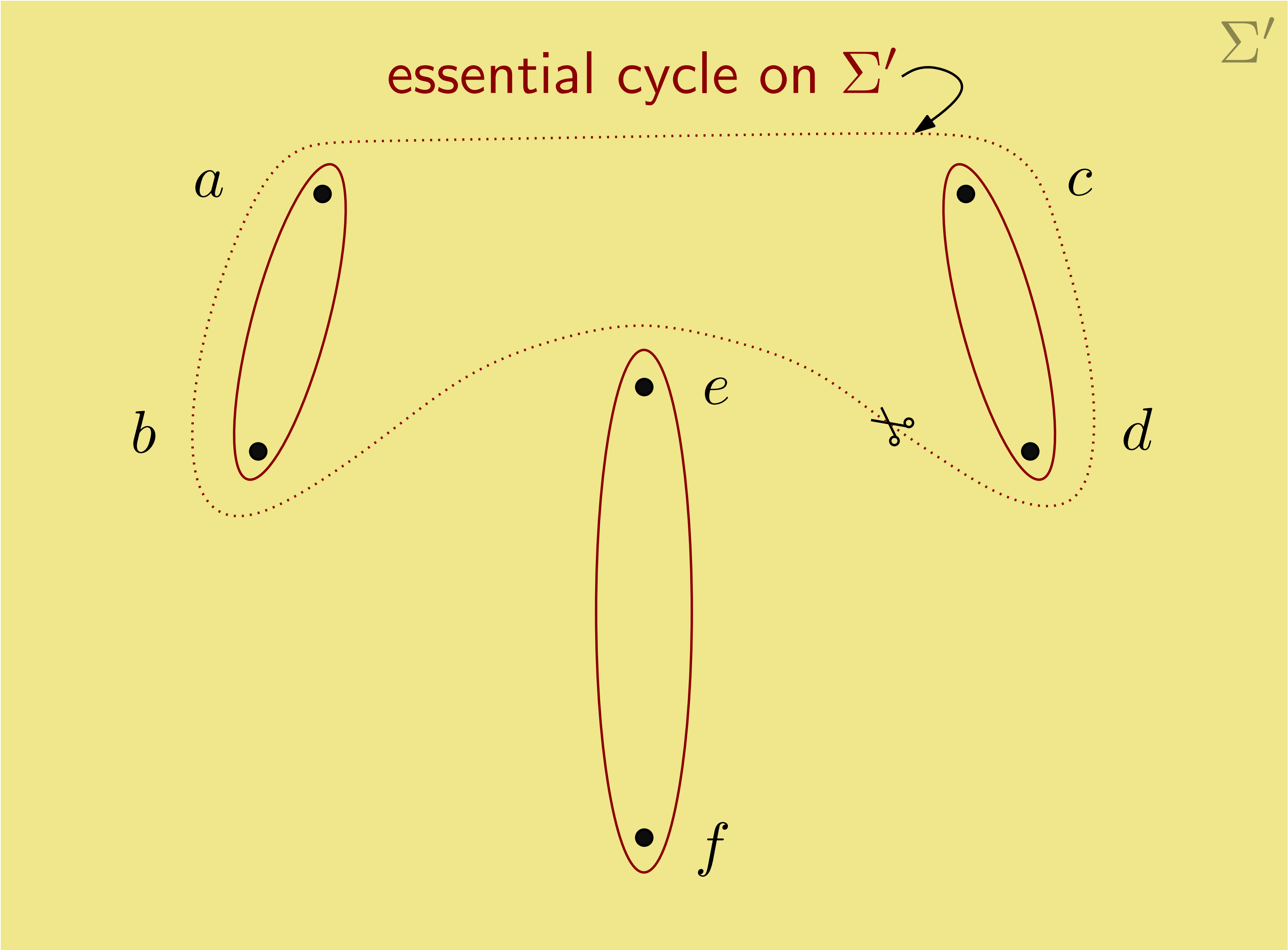

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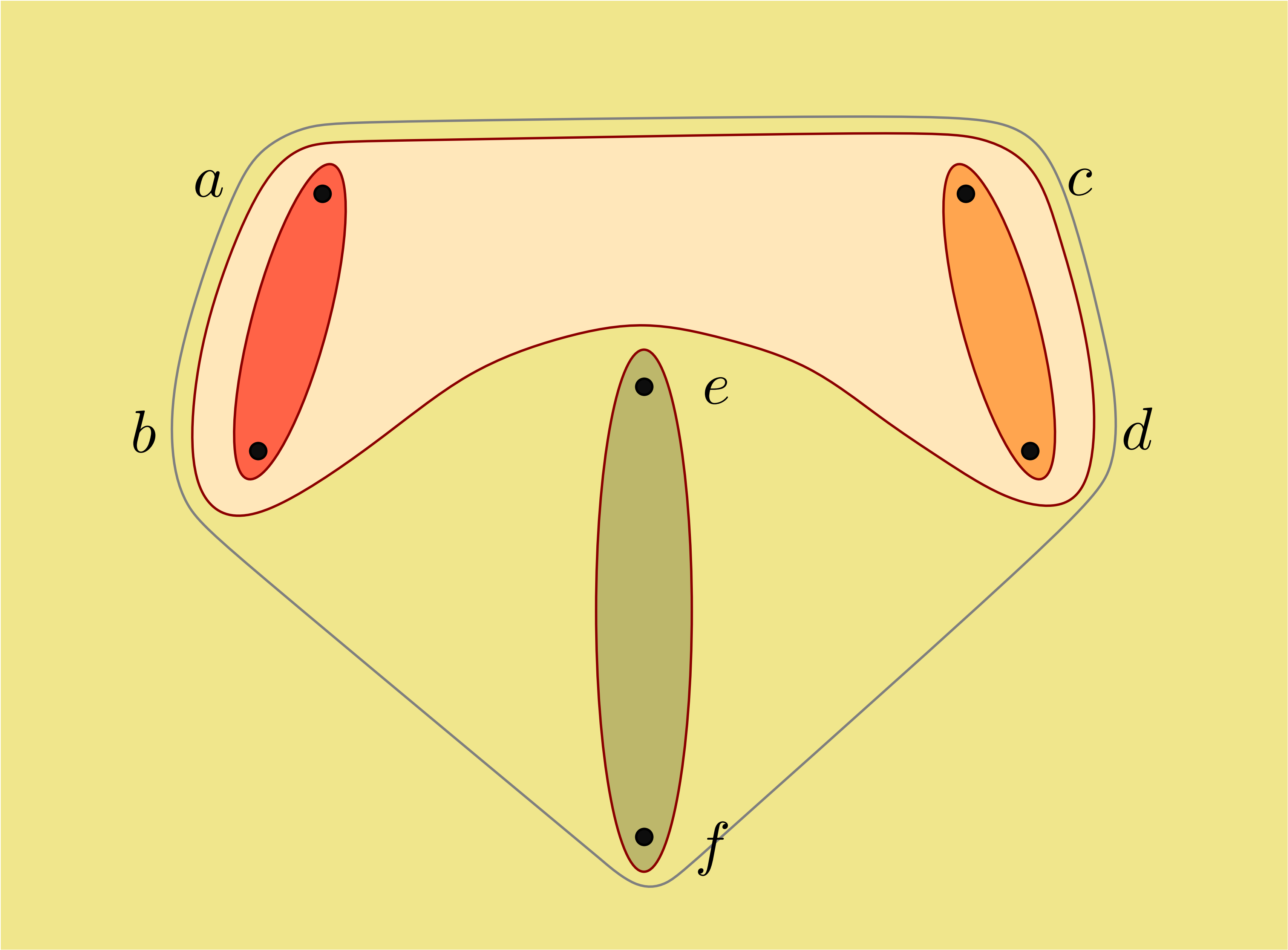
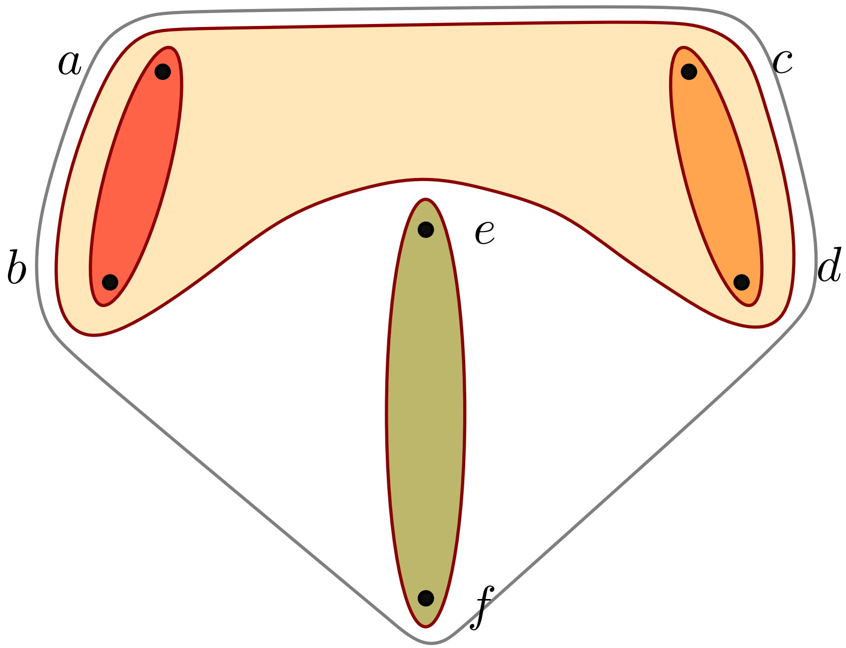
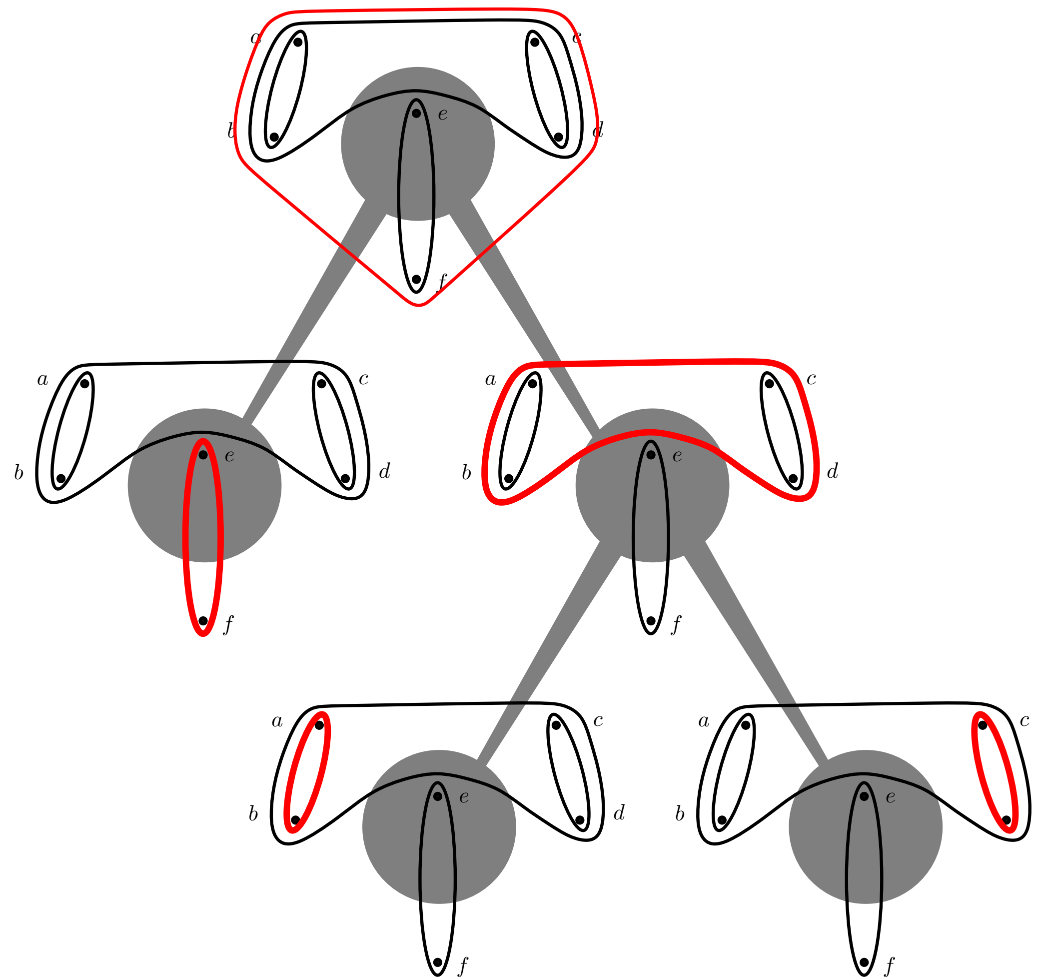


Properties





Simple closed curves

$n - 1$ non-crossing cycles

Nested in a binary tree

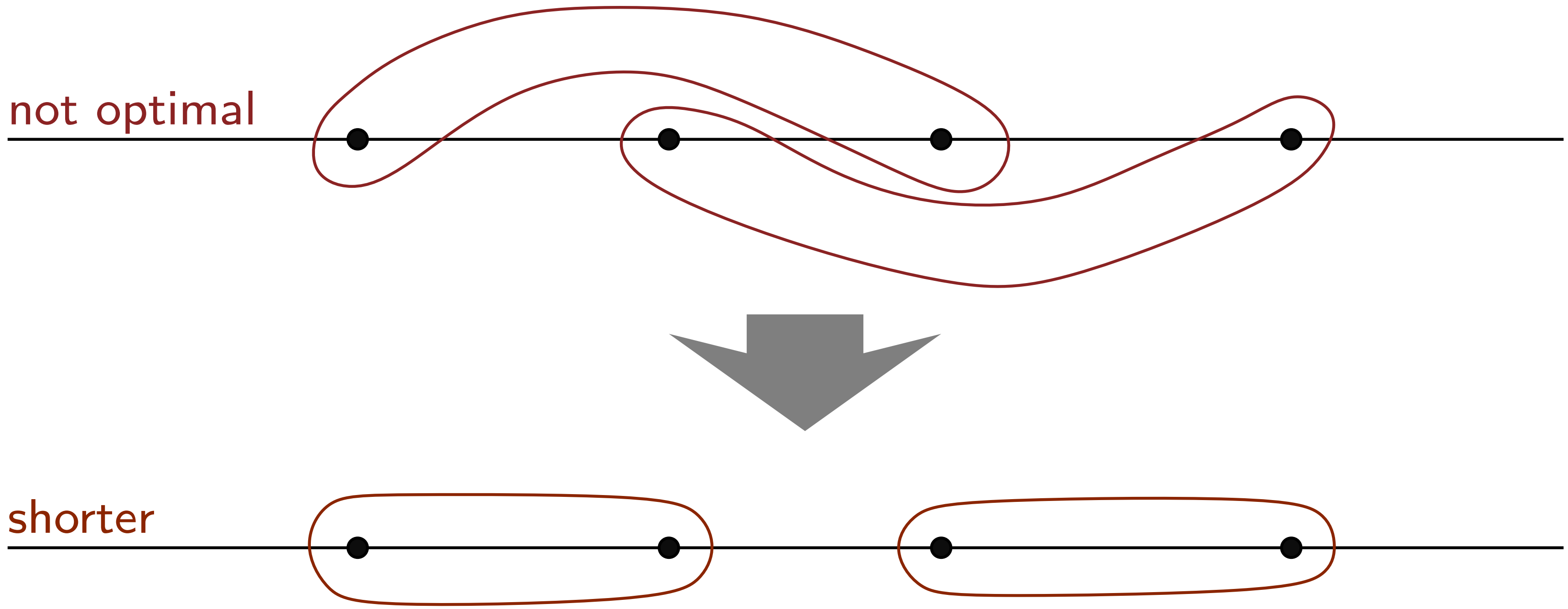
Shortest pants decomposition

Input: A set P of n points in the plane $\mathbb{E}^2$

$\Sigma = \mathbb{E}^2 \setminus P$

Problem: Compute a pants decomposition Π of Σ of minimum total length, i.e., the sum of the Euclidean lengths of the cycles in Π is the minimum

Points on a line



Lemma: Every cycle in a shortest pants decomposition of collinear points encloses an *interval* of points

Compute shortest pants decomposition in $O(n^2)$ time using *dynamic programming* with Yao's speedup

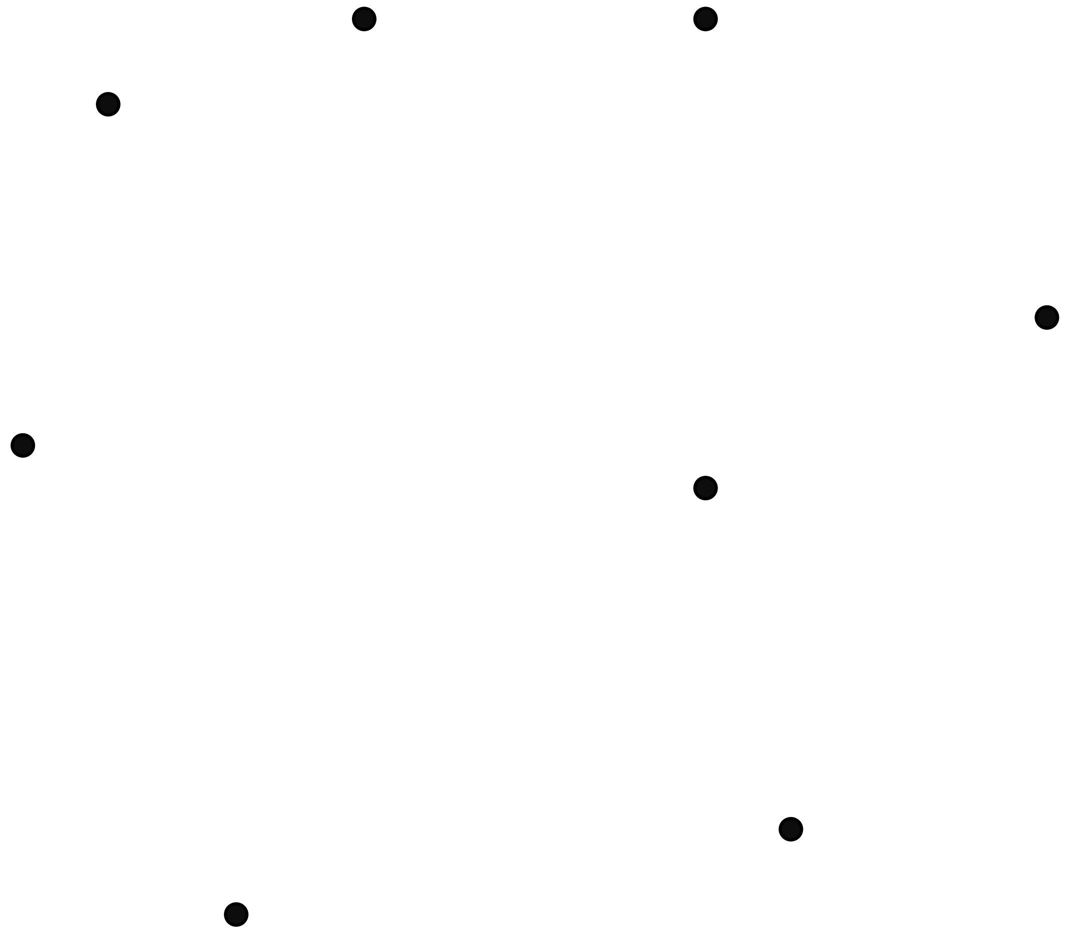
A lower bound

Every cycle in a shortest pants decomposition is a simple polygon with vertices in P
(no Steiner points)

A shortest pants decomposition Π^* of $\mathbb{E}^2 \setminus P$ contains a TSP tour of P

So, $|\Pi^*| \geq |TSP(P)|$

$O(\log n)$ approximation



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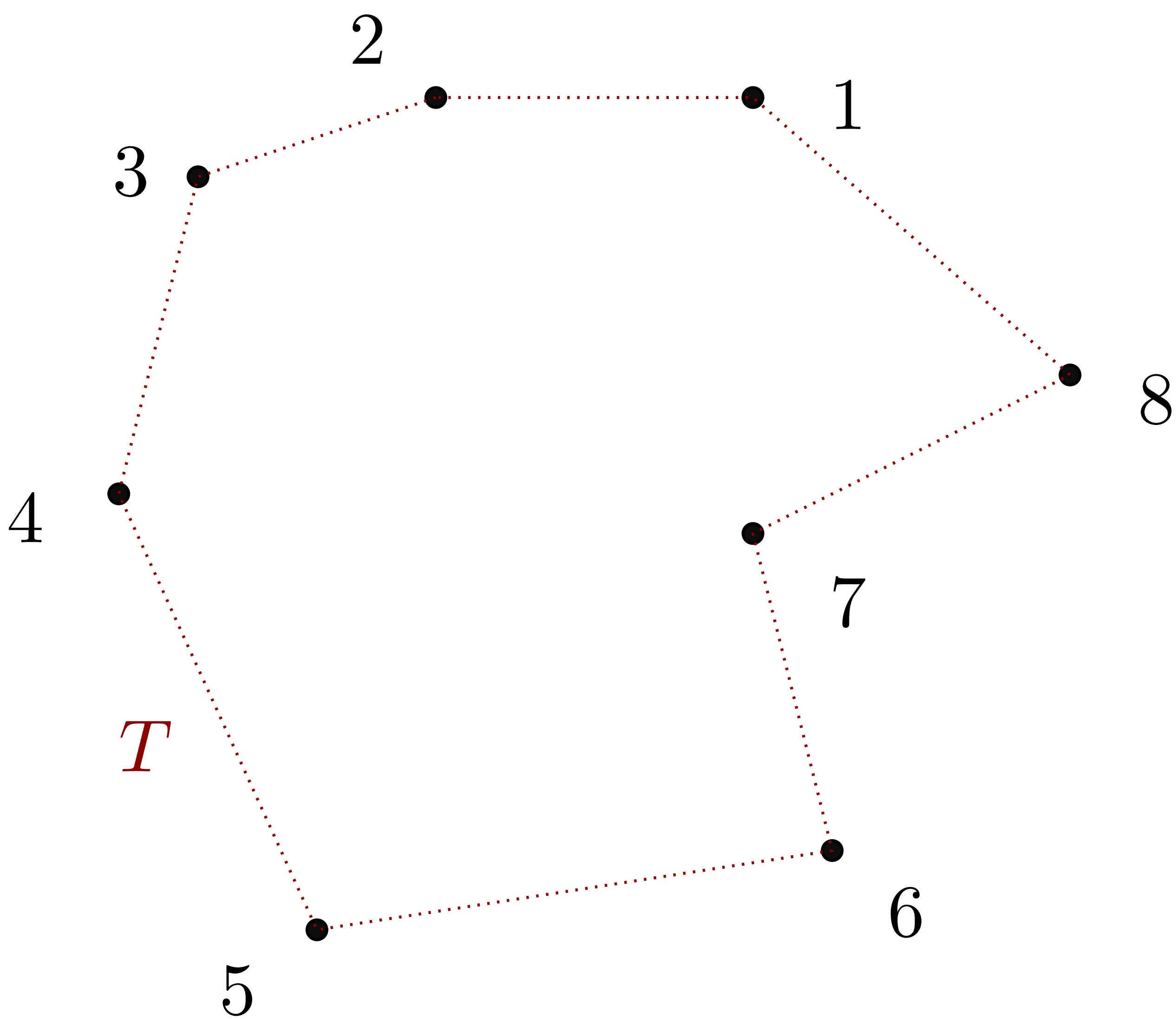
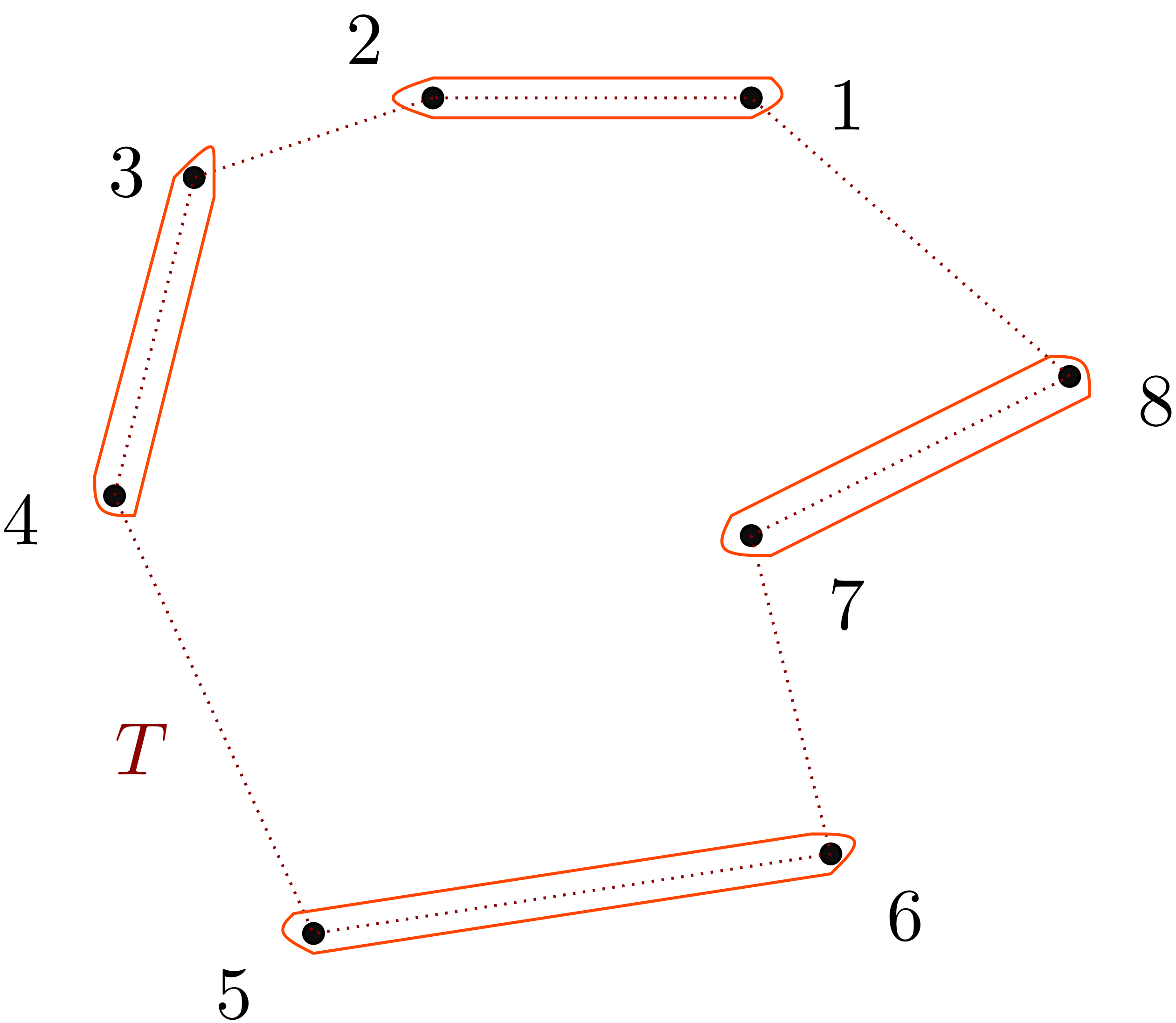


Construct an $O(1)$-approximate TSP tour T

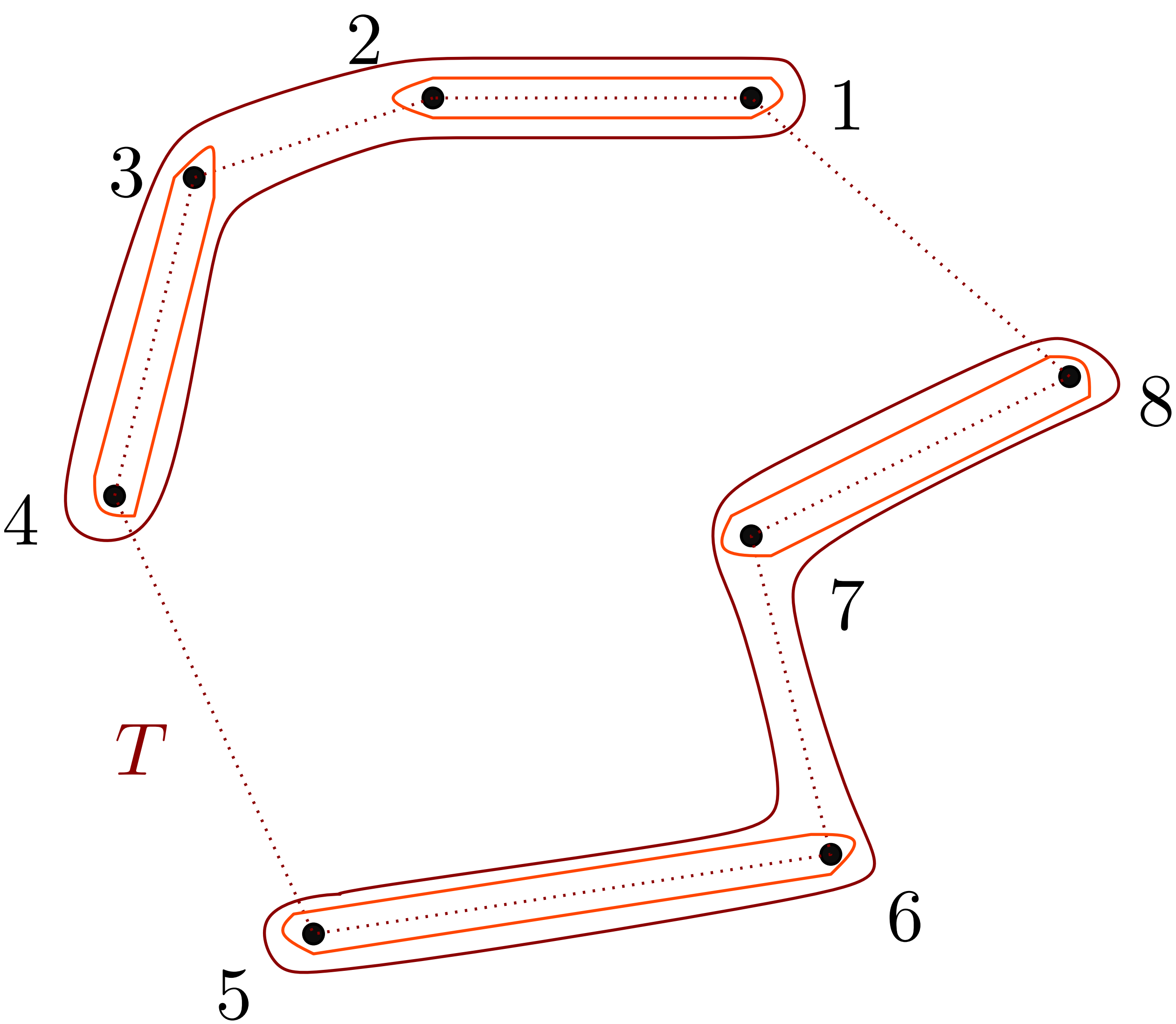
Start with the n points in order along the tour T

$O(\log n)$ approximation



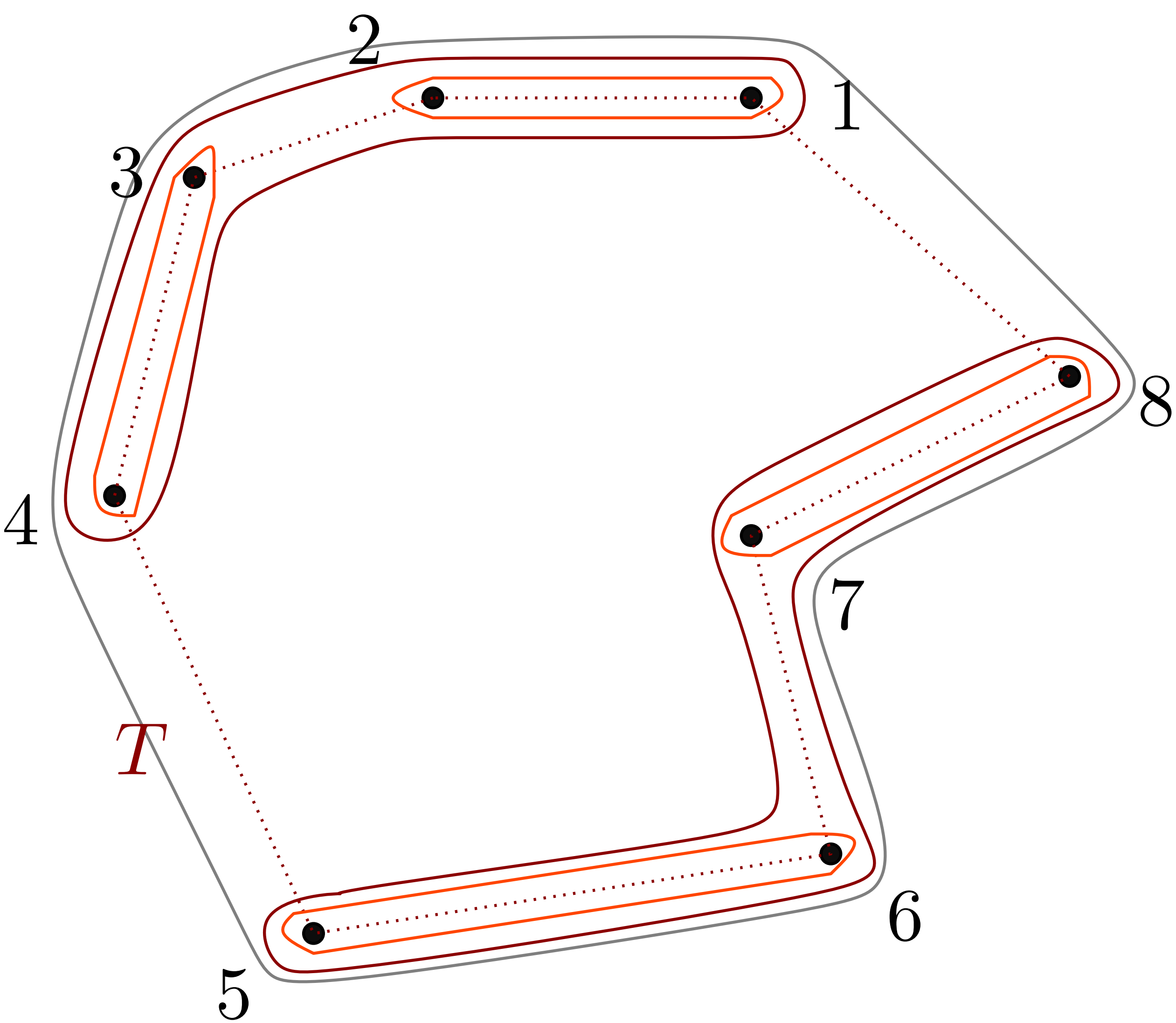
Repeatedly enclose pairs of smaller cycles by a bigger cycle until we have a pants decomposition Π

$O(\log n)$ approximation



Each cycle of Π is obtained by doubling the edges of a sub-tour of T

$O(\log n)$ approximation



Each edge of T belongs to $O(\log n)$ cycles of Π

So,

$$|\Pi| \leq O(\log n)\,|T| \\ \leq O(\log n)\,|\Pi^*|$$

PTAS

For every $\varepsilon > 0$, compute a $(1+\varepsilon)$-approximate shortest pants decomposition in polynomial time

Extension of PTAS for Euclidean TSP

Uses Mitchell's guillotine rectangular subdivisions

Efharisto!

Related work

Éric Colin de Verdière and Francis Lazarus.
Optimal Pants Decompositions and Shortest Homotopic Cycles on an Orientable Surface.
Graph Drawing, pp. 478–490, 2003 (+EuroCG'03)

Show how to *shorten* a given pants decomposition

Given a pants decomposition of a general combinatorial surface, they compute a *homotopic* pants decomposition in which each cycle is shortest in its homotopy class

Work in progress

NP-complete for general surfaces?

. . . I believe so!

NP-complete for the punctured plane?

. . . I don't know

An $O(1)$-approximation for the punctured plane should be possible using well-separated pairs decomposition